# Natural Language Processing

Lecture 9—2/10/2015

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# Today

- More on HMMs
  - Review statistical POS tagging
  - 3 HMM problems and algorithms
    - Decoding (Viterbi)
    - Forward/Backward
    - EM, (Forward-Backward or Baum-Welch)

# Getting to HMMs

- This equation gives us the best tag sequence

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}}\, P(t_1^n | w_1^n)$$

- But how to make it operational?
  - How to efficiently perform this computation?
- Intuition of Bayesian inference:
  - Use Bayes rule to transform this equation into a generative model

# Using Bayes Rule

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

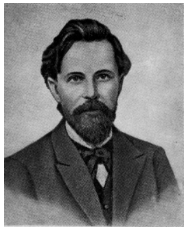
$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} \frac{P(w_1^n|t_1^n)P(t_1^n)}{P(w_1^n)}$$

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(w_1^n|t_1^n)P(t_1^n)$$

# Likelihood and Prior

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} \overbrace{P(w_1^n|t_1^n)}^{\text{likelihood}} \overbrace{P(t_1^n)}^{\text{prior}}$$

$$P(w_1^n | t_1^n) \approx \prod_{i=1}^{n} P(w_i | t_i)$$



$$P(t_1^n) \approx \prod_{i=1}^{n} P(t_i|t_{i-1})$$

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n|w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} P(w_i|t_i) P(t_i|t_{i-1})$$

# Two Kinds of Probabilities

- Tag transition probabilities $p(t_i|t_{i-1})$
  - Determiners likely to precede adjs and nouns
    - That/DT flight/NN
    - The/DT yellow/JJ hat/NN
    - So we expect P(NN|DT) and P(JJ|DT) to be high, but P(DT|JJ) to be low
  - Compute P(NN|DT) by counting in a labeled corpus:

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i)}{C(t_{i-1})}$$

$$P(NN|DT) = \frac{C(DT, NN)}{C(DT)} = \frac{56,509}{116,454} = .49$$

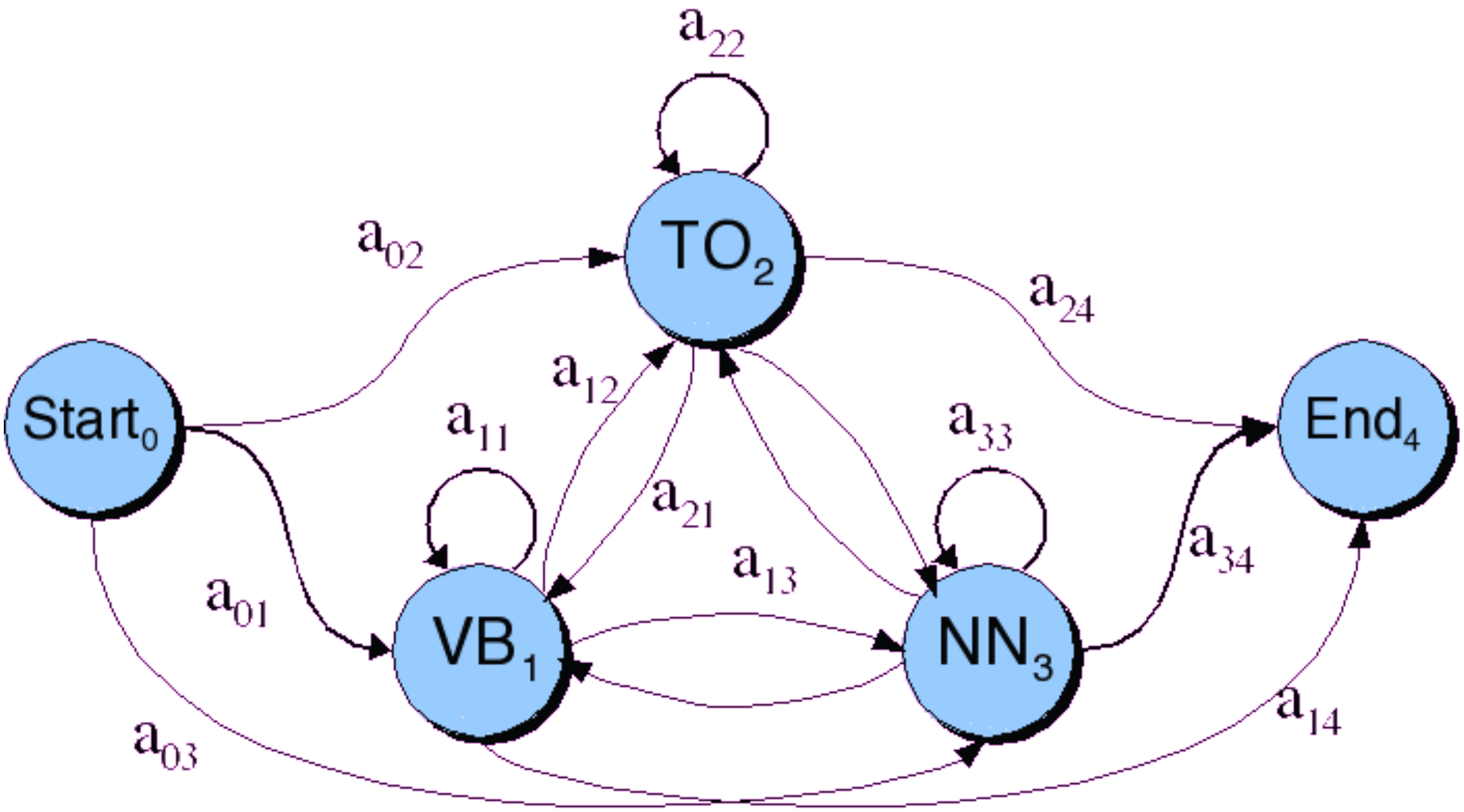
# Two Kinds of Probabilities

- Word likelihood probabilities $p(w_i|t_i)$
  - VBZ (3sg Pres verb) likely to be “is”
  - Compute P(is|VBZ) by counting in a labeled corpus:

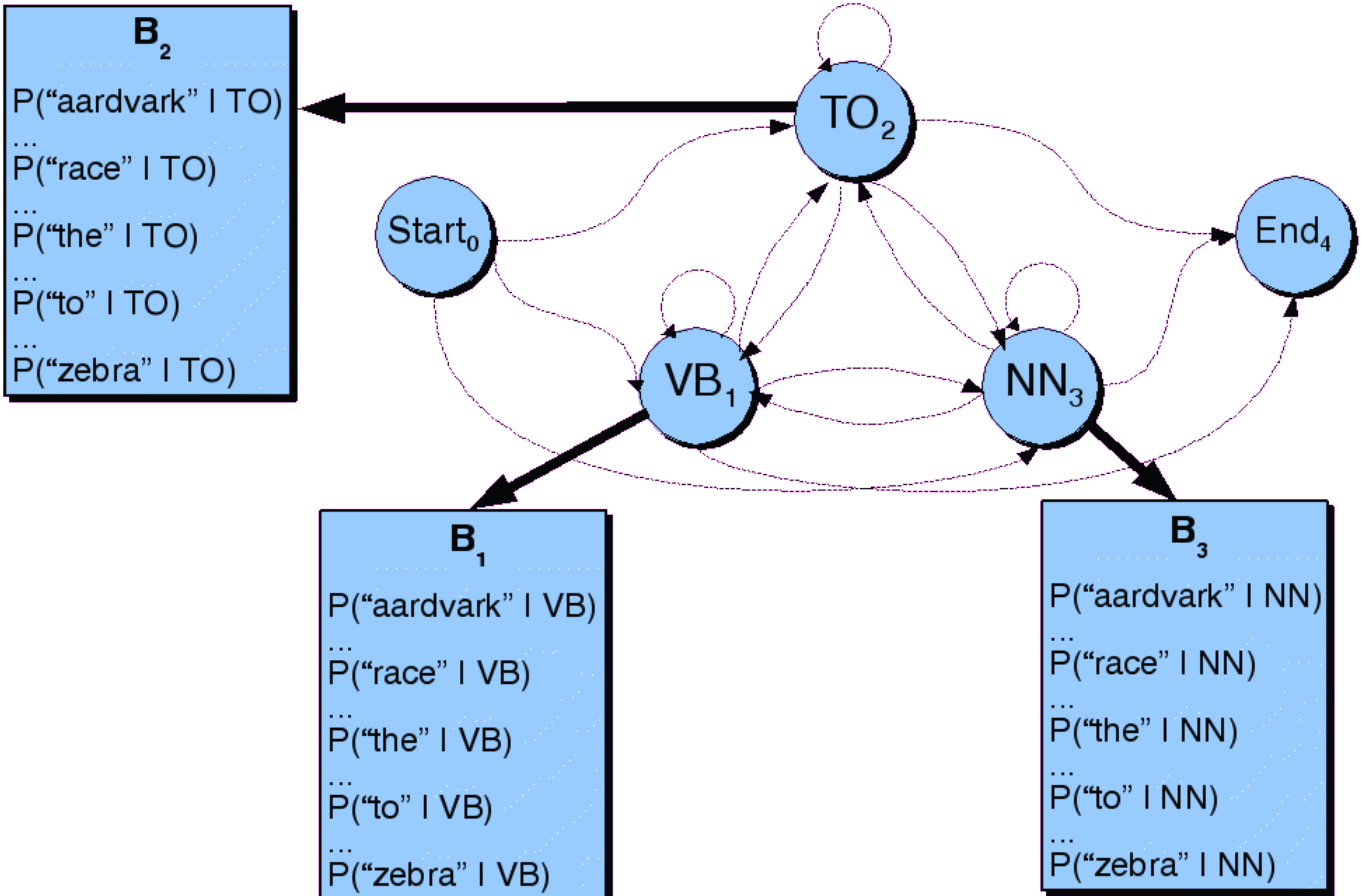
$$P(w_i|t_i) = \frac{C(t_i, w_i)}{C(t_i)}$$

$$P(is|VBZ) = \frac{C(VBZ, is)}{C(VBZ)} = \frac{10,073}{21,627} = .47$$

# Transition Probabilities



# Observation Likelihoods



# Question

- If there are 30 or so tags in the Penn set
- And the average sentence is around 20 words...
- How many tag sequences do we have to enumerate argmax over?

$30^{20}$

# Hidden Markov Models

- States $Q = q_1, q_2 \ldots q_{N;}$
- Observations $O = o_1, o_2 \ldots o_{N;}$
  - Each observation is a symbol from a vocabulary $V = \{v_1, v_2, \ldots v_V\}$
- Transition probabilities
  - Transition probability matrix $A = \{a_{ij}\}$

$$a_{ij} = P(q_t = j \mid q_{t-1} = i) \quad 1 \leq i, j \leq N$$

- Observation likelihoods
  - Output probability matrix $B = \{b_i(k)\}$

$$b_i(k) = P(X_t = o_k \mid q_t = i)$$

- Special initial probability vector $\pi$

$$\pi_i = P(q_1 = i) \quad 1 \leq i \leq N$$

# 3 Problems

- Given this framework there are 3 problems that we can pose to an HMM
  - Given an observation sequence, what is the probability of that sequence given a model?
  - Given an observation sequence and a model, what is the most likely state sequence?
  - Given an observation sequence, infer the best model parameters for a skeletal model

# Problem 1

- The probability of a sequence given a model...

**Computing Likelihood:** Given an HMM $\lambda = (A, B)$ and an observation sequence $O$, determine the likelihood $P(O|\lambda)$.

  - Used in model development... How do I know if some change I made to the model is making it better
  - And in classification tasks
    - Word spotting in ASR, language identification, speaker identification, author identification, etc.
      - Train one HMM model per class
      - Given an observation, pass it to each model and compute P(seq|model).

# Problem 2

- Most probable state sequence given a model and an observation sequence

**Decoding**: Given as input an HMM $\lambda = (A,B)$ and a sequence of observations $O = o_1, o_2, ..., o_T$, find the most probable sequence of states $Q = q_1 q_2 q_3 \dots q_T$.

  - Typically used in tagging problems, where the tags correspond to hidden states
    - As we' ll see almost any problem can be cast as a sequence labeling problem
  - Viterbi solves problem 2

# Problem 3

- Infer the best model parameters, given a skeletal model and an observation sequence...
  - That is, fill in the A and B tables with the right numbers...
    - The numbers that make the observation sequence most likely
  - Useful for getting an HMM without having to hire annotators...
    - That is you tell me how many tags there are and give me a boatload of untagged text, and I give you back a part of speech tagger.

# Solutions

- Problem 2: Viterbi
- Problem 1: Forward
- Problem 3: Forward-Backward
  - An instance of EM

# Problem 2: Decoding

- Ok, now we have a complete model that can give us what we need. Recall that we need to get

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n | w_1^n)$$

- We could just enumerate all paths given the input and use the model to assign probabilities to each.
  - Not a good idea.
  - Luckily dynamic programming helps us here

# Intuition

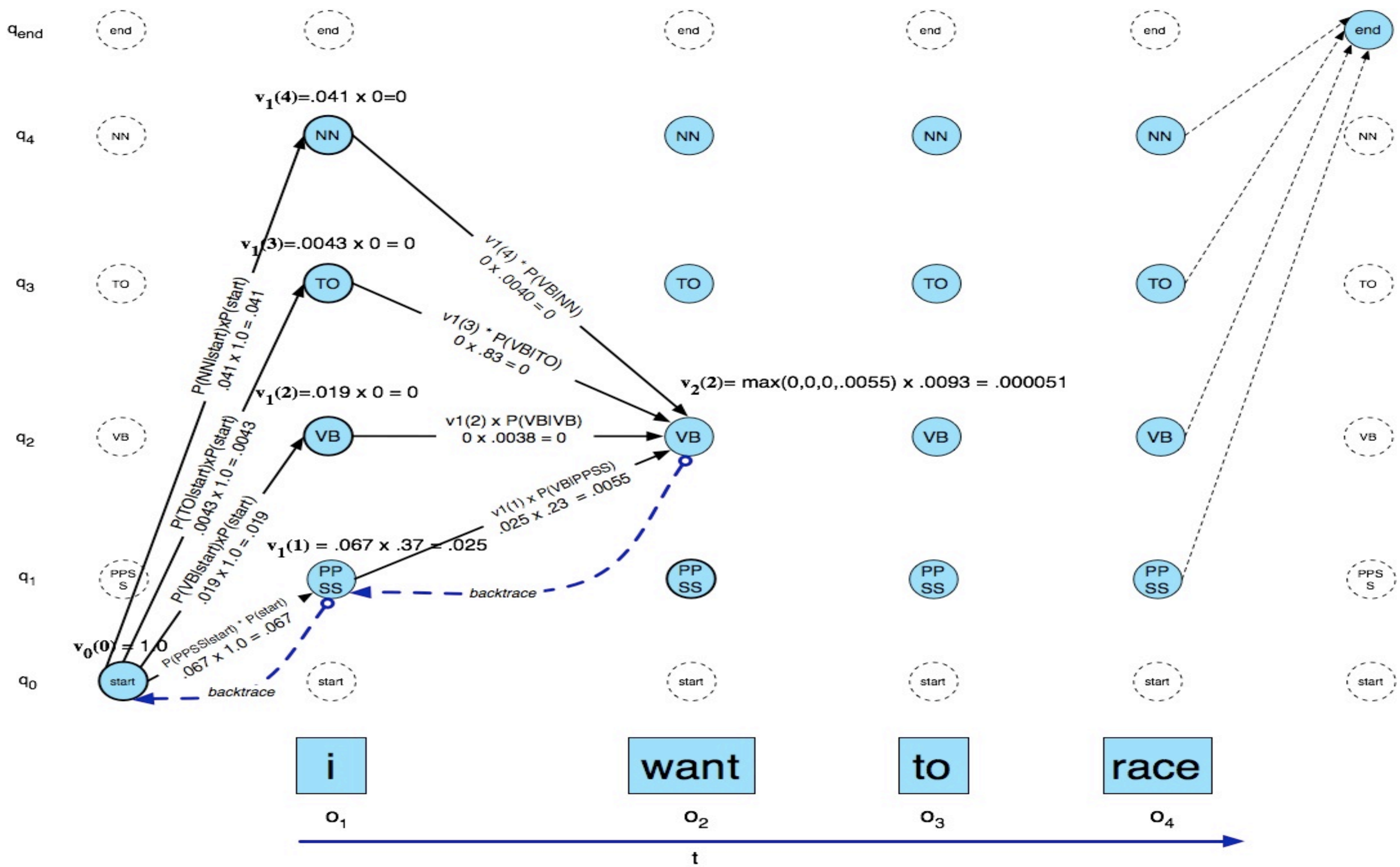
- You’re interested in the shortest distance from Boulder to Moab
- Consider a possible location on the way to Moab, say Glenwood Springs.
- What do you need to know about all the different possible ways to get to Glenwood Springs?

The best way (the shortest path)

# Intuition

- Consider a state sequence (tag sequence) that ends at state j (i.e., has a particular tag T at the end)
- The probability of that tag sequence can be broken into parts
  - The probability of the BEST tag sequence up through j-1
  - Multiplied by the transition probability from the tag at the end of the j-1 sequence to T.
  - And the observation probability of the observed word given tag T

# Viterbi Example



# The Viterbi Algorithm

**function** VITERBI(*observations* of len $T$,*state-graph* of len $N$) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*
**for** each state $s$ **from** 1 **to** $N$ **do** ; initialization step
$viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
$backpointer[s,1] \leftarrow 0$
**for** each time step $t$ **from** 2 **to** $T$ **do** ; recursion step
**for** each state $s$ **from** 1 **to** $N$ **do**
$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$
$$backpointer[s,t] \leftarrow \operatorname*{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$
$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$ ; termination step
$$backpointer[q_F,T] \leftarrow \operatorname*{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$ ; termination step
**return** the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

# Viterbi Summary

- Create an array
  - With columns corresponding to inputs
  - Rows corresponding to possible states
- Sweep through the array in one pass filling the columns left to right using our transition probs and observations probs
- Dynamic programming key is that we need only store the MAX prob and path to each cell, (not all paths).

# Evaluation

- So once you have you POS tagger running how do you evaluate it?
  - Overall error rate with respect to a gold-standard test set
    - Each token gets a tag, so overall accuracy is a decent measure (number correct/number tagged)
  - But to improve a system we want more detailed information
    - Per word accuracy
    - Confusion matrices

# Evaluation

- Results are compared with a manually coded “Gold Standard”
  - Typically accuracy reaches 96-97%
  - This may be compared with result for a baseline tagger (one that uses no context)
- Important: 100% accuracy is impossible even for human annotators
  - Goal is to get system performance near to human performance
    - Beware of claims from systems that claim to exceed the accuracy of human annotators

# Detailed Error Analysis

- Look at a confusion matrix

| | IN | JJ | NN | NNP | RB | VBD | VBN |
|---|---|---|---|---|---|---|---|
| IN | — | .2 | | | .7 | | |
| JJ | .2 | — | 3.3 | 2.1 | 1.7 | .2 | 2.7 |
| NN | | 8.7 | — | | | | .2 |
| NNP | .2 | 3.3 | 4.1 | — | .2 | | |
| RB | 2.2 | 2.0 | .5 | | — | | |
| VBD | | .3 | .5 | | | — | 4.4 |
| VBN | | 2.8 | | | | 2.6 | — |

- See what errors are causing problems
  - Noun (NN) vs ProperNoun (NNP) vs Adj (JJ)
  - Preterite (VBD) vs Participle (VBN) vs Adjective (JJ)

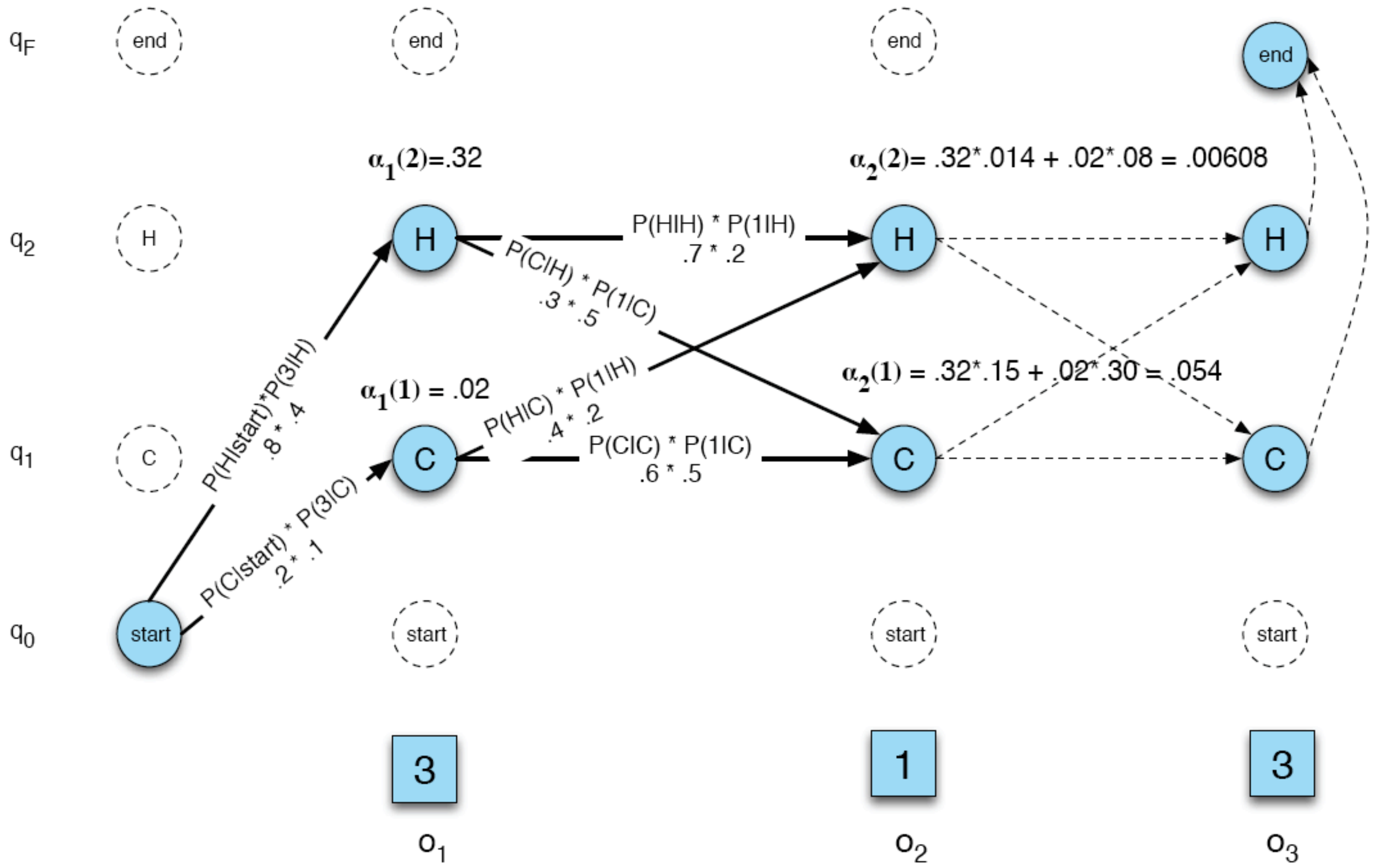
# Problem 1: Forward

- Given an observation sequence return the probability of the sequence given the model...
  - Well in a normal Markov model, the states and the sequences are identical... So the probability of a sequence is the probability of the path sequence
  - But not in an HMM... Remember that any number of sequences might be responsible for any given observation sequence.

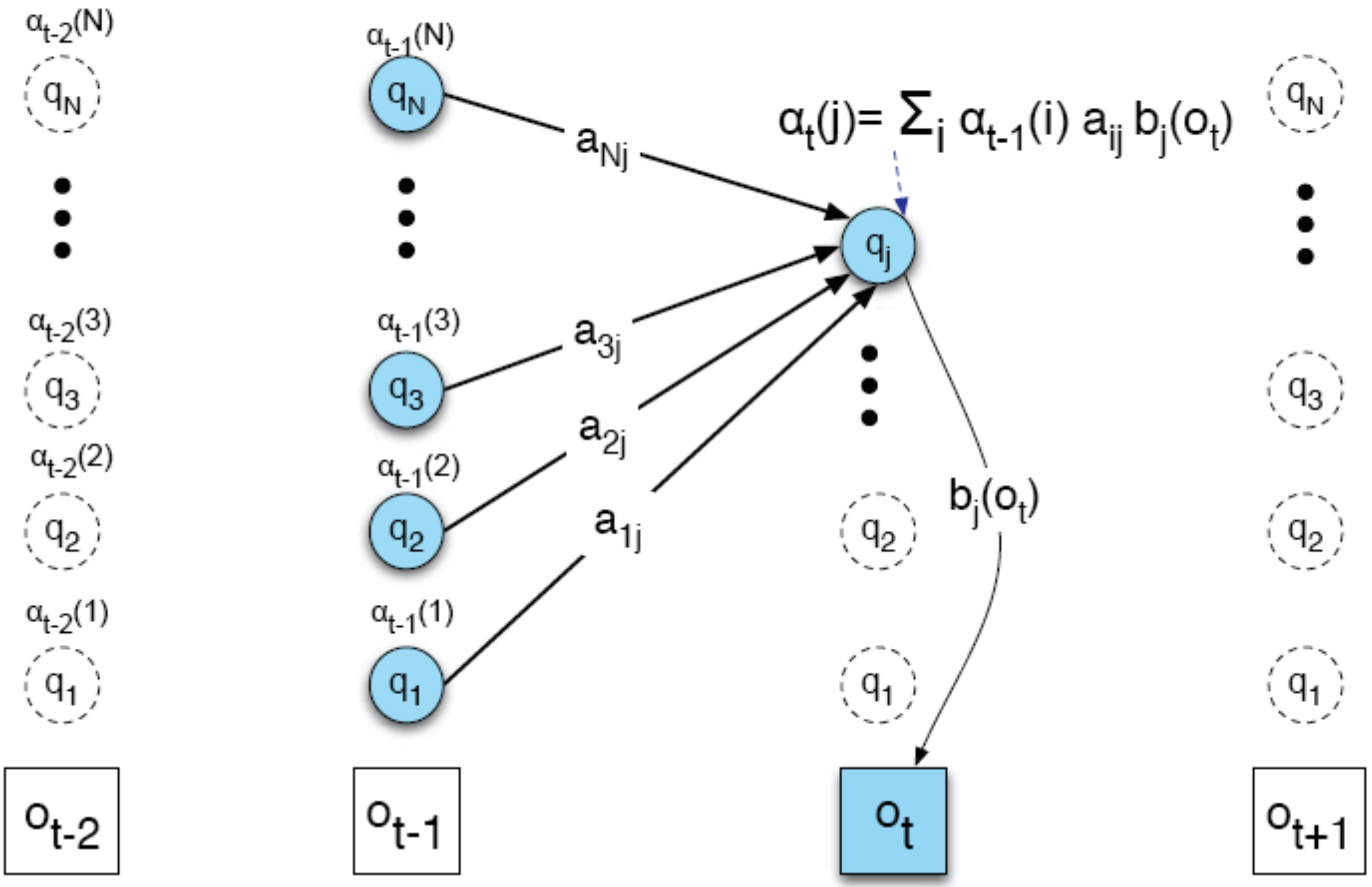
# Forward

- Efficiently computes the probability of an observed sequence given a model
  - P(sequence|model)
- Nearly identical to Viterbi; replace the MAX with a SUM

# Ice Cream Example



# Ice Cream Example



# Forward

**function** FORWARD(*observations* of len $T$, *state-graph* of len $N$) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
**for** each state $s$ **from** 1 **to** $N$ **do** ; initialization step
$forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
**for** each time step $t$ **from** 2 **to** $T$ **do** ; recursion step
**for** each state $s$ **from** 1 **to** $N$ **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$ ; termination step

**return** $forward[q_F,T]$

# Problem 3: Learning the Parameters

- First an example to get the intuition down
- We'll do Forward-Backward next time

# Urn Example

- A genie has two urns filled with red and blue balls. The genie selects an urn and then draws a ball from it (and replaces it). The genie then selects either the same urn or the other one and then selects another ball…
  - The urns and actual draws are hidden
  - The balls are observed

# Urn

- Based on the results of a long series of draws...
  - Figure out the distribution of colors of balls in each urn
    - Observation probabilities (B table)
  - Figure out the genie’s preferences for going from one urn to the next
    - Transition probabilities (A table)

# Urns and Balls

- Pi: Urn 1: 0.9; Urn 2: 0.1
- A

| | Urn 1 | Urn 2 |
|---|---|---|
| Urn 1 | 0.6 | 0.4 |
| Urn 2 | 0.3 | 0.7 |

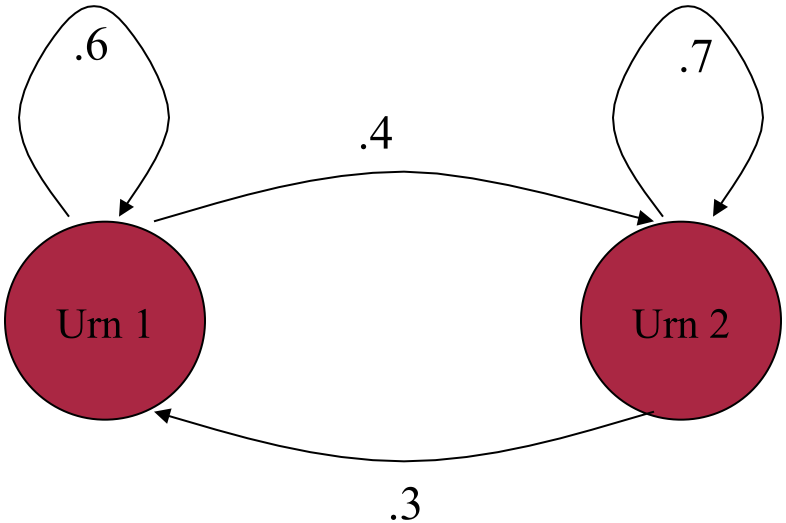
- B

| | Urn 1 | Urn 2 |
|---|---|---|
| Red | 0.7 | 0.4 |
| Blue | 0.3 | 0.6 |

# Urns and Balls

- Let's assume the input (observables) is Blue Blue Red (BBR)
- Since both urns contain red and blue balls any path of length 3 through this machine could produce this output

How many paths are there?



# Urns and Balls

Blue Blue Red

| | |
|---|---|
| 1 1 1 | (0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204 |
| 1 1 2 | (0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077 |
| 1 2 1 | (0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136 |
| 1 2 2 | (0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181 |

| | |
|---|---|
| 2 1 1 | (0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052 |
| 2 1 2 | (0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020 |
| 2 2 1 | (0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052 |
| 2 2 2 | (0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070 |

# Urns and Balls

Viterbi: Says 111 is the most likely state sequence

| | |
|---|---|
| 1 1 1 | (0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204 |
| 1 1 2 | (0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077 |
| 1 2 1 | (0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136 |
| 1 2 2 | (0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181 |

| | |
|---|---|
| 2 1 1 | (0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052 |
| 2 1 2 | (0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020 |
| 2 2 1 | (0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052 |
| 2 2 2 | (0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070 |

# Urns and Balls

Forward: P(BBR| model) = .0792 $\sum$

| | |
|---|---|
| 1 1 1 | (0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204 |
| 1 1 2 | (0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077 |
| 1 2 1 | (0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136 |
| 1 2 2 | (0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181 |

| | |
|---|---|
| 2 1 1 | (0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052 |
| 2 1 2 | (0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020 |
| 2 2 1 | (0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052 |
| 2 2 2 | (0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070 |

# Urns and Balls

- EM
  - What if I told you I lied about the numbers in the model (Priors,A,B) for this example? That is, I just made them up.
  - Can I get better numbers just from the input sequence?

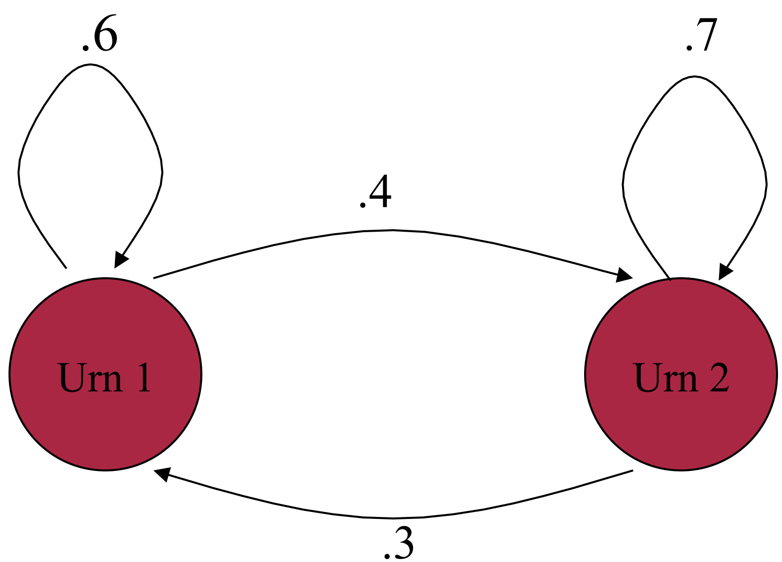
# Urns and Balls

- Yup
  - Just count up and prorate the number of times a given transition is traversed while processing the observations inputs.
  - Then use that pro-rated count to re-estimate the transition probability for that transition

# Urns and Balls

- But… we just saw that don't know the actual path the input took, its hidden!
  - So prorate the counts from all the possible paths based on the path probabilities the model gives you
    - Basically do what Forward does
- But you said the numbers were wrong
  - Doesn't matter; use the original numbers then replace the old ones with the new ones.

# Urn Example




Let's re-estimate the Urn1->Urn2 transition and the Urn1->Urn1 transition (using Blue Blue Red as training data).

# Urns and Balls

Blue Blue Red

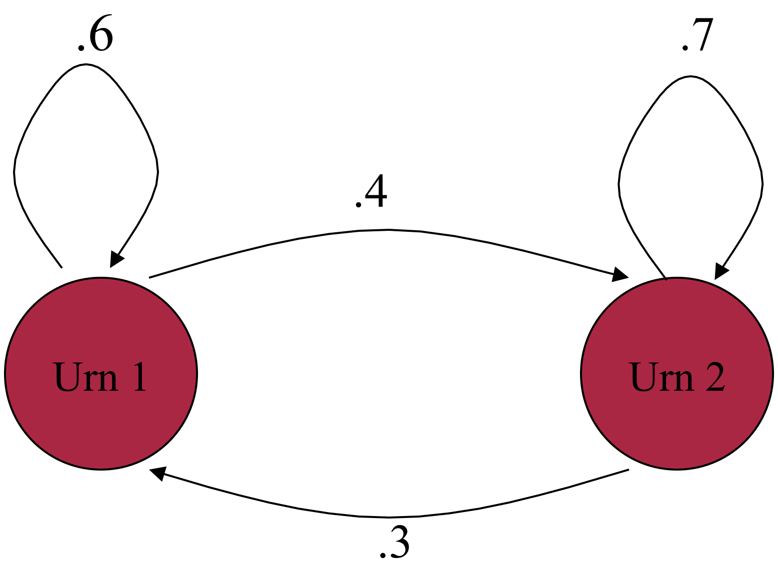
| | |
|---|---|
| 1 1 1 | (0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204 |
| 1 1 2 | (0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077 |
| 1 2 1 | (0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136 |
| 1 2 2 | (0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181 |

| | |
|---|---|
| 2 1 1 | (0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052 |
| 2 1 2 | (0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020 |
| 2 2 1 | (0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052 |
| 2 2 2 | (0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070 |

# Urns and Balls

- That's
  - (.0077*1)+(.0136*1)+(.0181*1)+(.0020*1)
  = .0414
- Of course, that's not a probability, it needs to be divided by the probability of leaving Urn 1 total.
- There's only one other way out of Urn 1 (going back to urn1)
  - So let's reestimate Urn1-> Urn1

# Urn Example




Let's re-estimate the Urn1->Urn1 transition

# Urns and Balls

Blue Blue Red

| | |
|---|---|
| 1 1 1 | (0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204 |
| 1 1 2 | (0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077 |
| 1 2 1 | (0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136 |
| 1 2 2 | (0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181 |

| | |
|---|---|
| 2 1 1 | (0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052 |
| 2 1 2 | (0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020 |
| 2 2 1 | (0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052 |
| 2 2 2 | (0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070 |

# Urns and Balls

- That's just
  - (2*.0204)+(1*.0077)+(1*.0052) = .0537
- Again not what we need but we're closer… we just need to normalize using those two numbers.

# Urns and Balls

- The 1->2 transition probability is
  .0414/(.0414+.0537) = 0.435
- The 1->1 transition probability is
  .0537/(.0414+.0537) = 0.565
- So in re-estimation the 1->2 transition went from .4 to .435 and the 1->1 transition went from .6 to .565

# EM Re-estimation

- Not done yet. No reason to think those values are right. *But they're more right than they used to be.*
  - So do it again, and again and....
- As with Problems 1 and 2, you wouldn't actually compute it this way. The Forward-Backward algorithm re-estimates these numbers in the same dynamic programming way that Viterbi and Forward do.