Logical Representations

LING 7800/CSCI 7000

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The Semantic Wall

Physical Symbol System

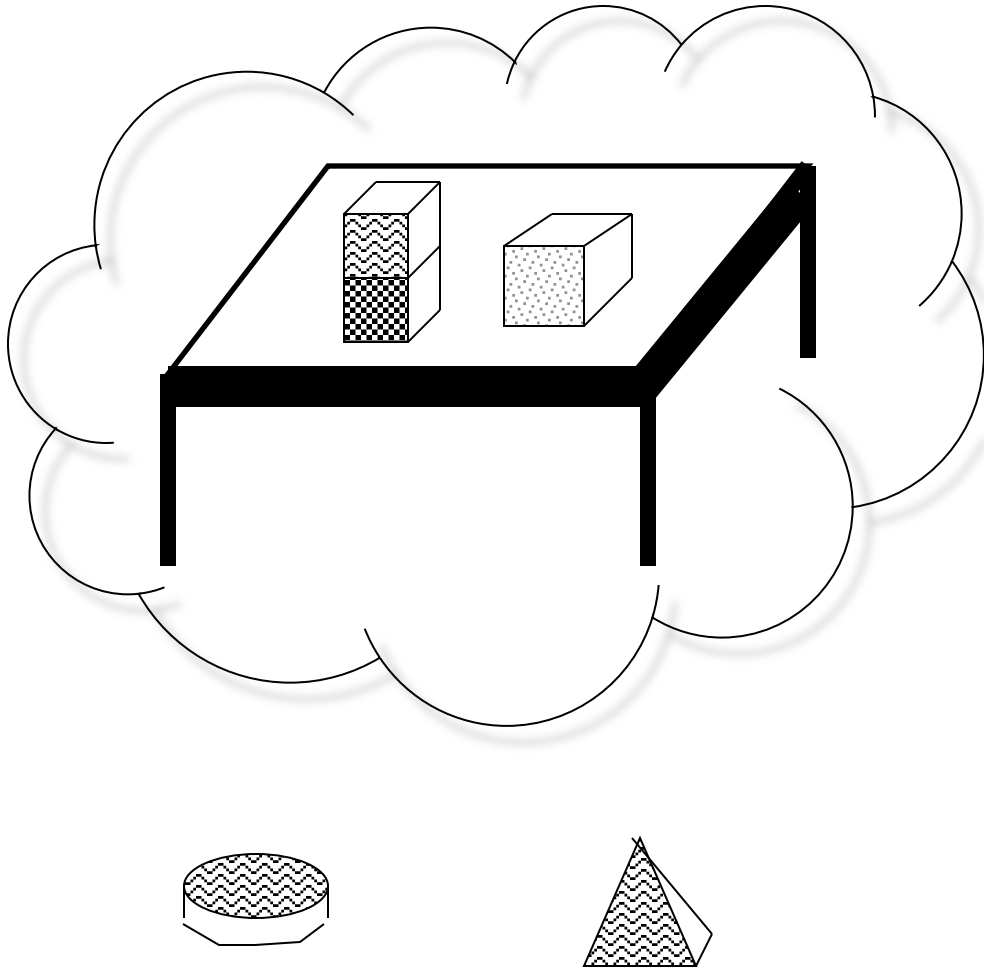
+BLOCKA+

+BLOCKB+

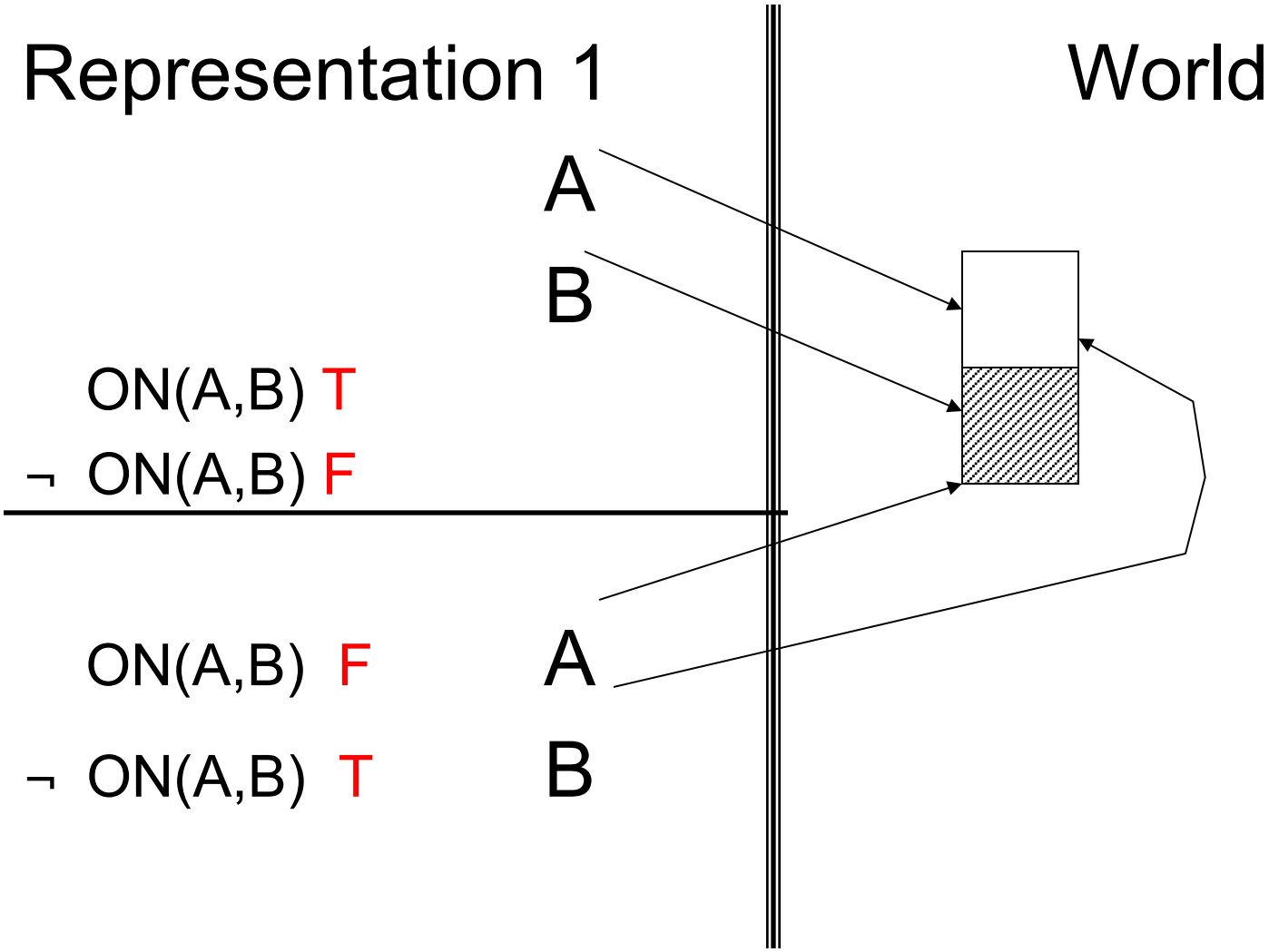
+BLOCKC+

P_1:(IS_ON +BLOCKA+ +BLOCKB+)
P_2:((IS_RED +BLOCKA+)

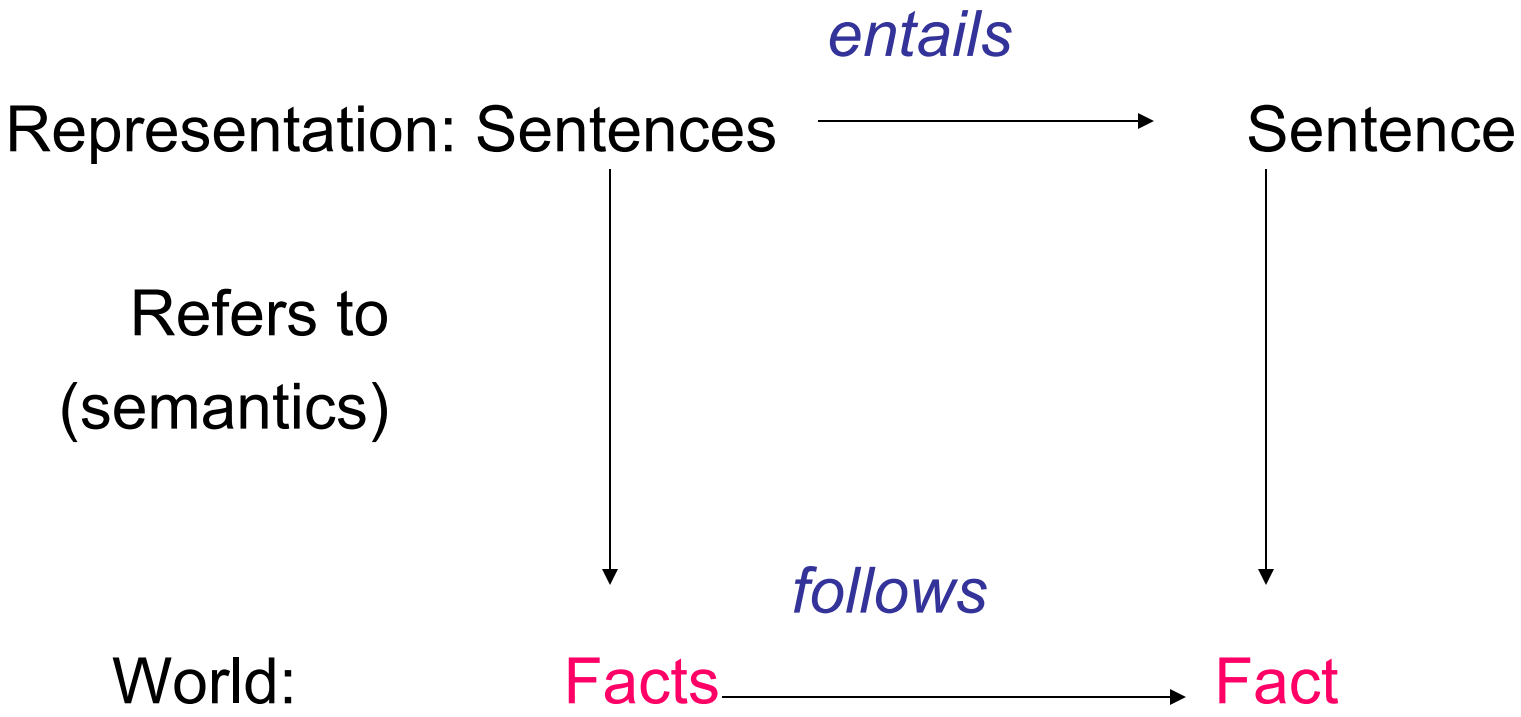
World



Truth depends on Interpretation



Logical Interpretations



Logic

- Truth.
  - A sentence is true if the *state of affairs* it describes is actually the case in the world.
- Valid.
  - A sentence is **valid** if it's true no matter what the world is actually like or what the semantics is.
  - *tautologies* or *analytic truths*.

Two KR Languages

Representation Language

- Propositional Logic
- First Order Logic

Inference Procedure

- Unit Resolution
- General Resolution

(Basis for Prolog)

Propositional Logic

- **Syntax.**
  - Sentences
    - T – True, F – False
    - P, Q, etc.
    - (Sentence), Sentence Connective Sentence, Sentence
  - Connectives
    - ¬ negation,
    - ∧ conjunction,
    - ∨ disjunction,
    - → implication, also (⇒,⊃)
    - ⇔ equivalence (biconditional implication)

Truth Tables

P: *The sun is shining*

Q: *The day is warm*

R: *The sun is shining and the day is warm*.

P	Q	P ∧ Q
F	F	
F	T	
T	F	
T	T	

Propositional Logic

- **Semantics.** Specifies what facts in the world sentences refer to.

  P. Richard is a pacifist, **Q.** Richard is a Quaker.

  Truth Table gives an *interpretation*

P	Q	¬P	P /\ Q	P \/ Q	P → Q
F	F	T	F	F	T
F	T	T	F	T	T
T	F	F	F	T	F
T	T	F	T	T	T

Unit Clauses

- $P \rightarrow Q$ is equivalent to $\neg P \vee Q$

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$\neg P \vee Q$
F	F	T	F	F	T	
F	T	T	F	T	T	
T	F	F	F	T	F	
T	T	F	T	T	T	

Flourishes

- Exclusive OR
  - *You will pay the fine or you will go to jail*

- Bi-conditionals – If and only if
  - $P \rightarrow Q \ \wedge \ Q \rightarrow P$; $P \rightarrow Q \ \wedge P \leftarrow Q$
  - *A triangle is isosceles **if and only if** the triangle has two congruent (equal) sides.*
  - *We'll leave **if and only if** we're forced to.*

Flourishes, cont.

- Inclusive \/; exclusive \/
  - *We spend the afternoons swimming or sunbathing.*
  - *They can resuscitate him or allow him to die.*
  - *If the site is in a particularly sensitive area, or there are safety considerations, we can refuse planning permission.*

Linguistic intuitions about logic?

- *If Patricia goes to the party then Emmet will go too.*
- *If wishes were money then we'd all be rich.*
- *If I were an ostrich, then I would be a bird.*
- *If I were an ostrich, then I would not be a bird.*

Linguistic intuitions about “and”

- **P ∧ Q equivalent to Q ∧ P?**
  - *He woke up and saw on TV that he had won the lottery.*
  - *Combine the egg yolks with water in a bowl and whisk the mixture until foamy.*
  - *He made two false starts and was disqualified from the race.*
  - *Move and I’ll shoot!*

Necessary/analytic truth of propositions (sentences)

- Logically true by virtue of the meaning of the terms
  - *Either the Buffs will beat Florida State or the Buffs will not beat Florida State.* **P \/ ¬ P**
  - *If the Buffs beat Florida State then Florida State loses to the Buffs.* **Reversal**
  - *All teams who win are teams.* **Quantifiers**
  - *The train will either arrive or it won't arrive.*

Entailment 4.6

- P entails Q when the truth of P guarantees the truth of Q, and the falsity of Q guarantees the falsity of P.

- Composite Truth Table for entailment:

P		Q
T	→	T
F	→	T or F
F	←	F
T or F	←	T

Entailment

- P entails Q when the truth of P guarantees the truth of Q, and the falsity of Q guarantees the falsity of P.

- Composite Truth Table for entailment:

P		Q
T	→	T
F	→	T or F
F	←	F
T or F	←	T

P	Q	P→ Q
F	F	T
F	T	T
T	F	F
T	T	T

Entailment VS. Bi-conditional

- $P \rightarrow Q$
- P's truth is sufficient for Q's truth
- Q's truth is necessary for P's truth

- $P \leftrightarrow Q$
- P's truth is necessary AND sufficient for Q's truth, and vice versa

Examples

- Q is necessary for P, P is sufficient for Q
  - Human → Mammal
  - Jump → Leave_ground
  - Prime (but not 2) → Odd
  - Senator → ≥ 30_years_old
- P is necessary and sufficient for Q, IFF
  - A person is a bachelor *iff* that person is a marriageable man who has never married.

Entailment Examples

- The anarchist assassinated the emperor. P
- The emperor died. Q
- $P \rightarrow Q$
- If P is true is Q true?
- If Q is False, is P false?
- If P is false, then what about Q?

Sources for Entailments

- Hyponyms
  - *I bought a dress today*. P
  - *I bought clothes today*. Q
- Syntax
  - *Caesar won the war*. P
  - *The war was won by Caesar*. Q
- Paraphrases – mutually entail each other

And now also….

- Necessarily true – tautologies
  - *Venus is Venus*
- Necessarily false – contradictions
  - *Here is not here*.
- Presuppositions
  - *The King of France is bald.*

    presupposes
  - There is a *King of France.*

Presuppositions

- *Her husband is a drunk.*

  She has a husband
- *I don't regret leaving Philadelphia*

  I left Philadelphia
- *The Prime Minister of Pakistan is in negotiations with the US.*

  Pakistan has a Prime Minister.

Presuppositions as a Truth Relation

- If P is true then Q (the presupposition) is true. If P is false then Q is still true. If Q is true, P could be true or false

- Composite Truth Table for presupposition:

P		Q
T	→	T
F	→	T
T or F	←	T

Entailment

- P entails Q when the truth of P guarantees the truth of Q, and the falsity of Q guarantees the falsity of P.

- Composite Truth Table for entailment:

P		Q
T	→	T
F	→	T or F
F	←	F
T or F	←	T

P	Q	P→ Q
F	F	T
F	T	T
T	F	F
T	T	T

Entailments compared to Presuppositions as Truth Relations

- Negate the entailing sentence, the entailment fails
  - *I saw my father today/I saw someone today.*
  - *I didn't see my father today/I didn't see someone today.*
- Negate a presupposing sentence, the presupposition still holds
  - *The mayor of Denver is/is not in Boulder.*

  Denver has a mayor

Unit Clauses

- $P \rightarrow Q$ is equivalent to $\neg P \vee Q$

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$\neg P \vee Q$
F	F	T	F	F	T	
F	T	T	F	T	T	
T	F	F	F	T	F	
T	T	F	T	T	T	

Inference Rules

- And Elimination

$$\frac{\alpha_1 \wedge \alpha_2, \wedge \ldots \wedge \alpha_n}{\alpha_1}$$

- And Introduction

$$\frac{\alpha_1, \ldots, \alpha_n}{\alpha_1 \wedge \alpha_2, \wedge \ldots \wedge \alpha_n}$$

Inference Rules (cont'd)

- Or Introduction

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2, \vee \ldots \vee \alpha_i \vee \ldots \vee \alpha_n}$$

- Double Negation Elimination

$$\frac{\neg \neg \alpha}{\alpha}$$

Inference Rules (cont'd)

- Modus Ponens (Implication Elimination)

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

De Morgan's laws

- ¬(P /\ Q) ↔ (¬P) \/ (¬Q)
  - "not (A and B)" is the same as "(not A) or (not B)"
  - The negation of a conjunction is the disjunction of the negations
- ¬(P \/ Q) ↔ (¬P) /\ (¬Q)
  - "not (A or B)" is the same as "(not A) and (not B)"
  - The negation of a disjunction is the conjunction of the negations.

Example

- Search A: NOT (cats OR dogs)
- Search B: (NOT cats) AND (NOT dogs)

  – Document 1: Contains only the word “cat(s)”.
  – Document 2: Contains only “dog(s)”.
  – Document 3: Contains both “cat(s)” and “dog(s)”.
  – Document 4: Contains neither “cat(s)” nor “dog(s)”.