Predicate Logic

LING 7430

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Comparing Logics

Ontology (ont = ‘to be’; logica = ‘word’): kinds of things one can talk about in the language

Examples:

- Propositional Logic — Facts
- Predicate Logic — Objects, Relationships among Objects
- Temporal Logics — Time Points or Intervals

Modeling Our World

- Propositions.
  - Alan-in-R225, Sam-in-R216, Jim-in….
  - R225-part of-Pender, R216-part of-Moore
- Limited.
  - Quickly gets explosive and cumbersome, can’t express generalizations
  - Can’t distinguish between objects and relations

Comparing Logics

Ontology (ont = ‘to be’; logica = ‘word’): kinds of things one can talk about in the language

Examples:

- Propositional Logic Facts
- **Predicate Logic Objects, Relationships among Objects**
- Temporal Logics Time Points or Intervals

Syntax of Predicate Logic

- Symbol set
  - **constants**
  - **Boolean connectives**
  - variables
  - functions
  - predicates (aka relations)
  - quantifiers
- Terms: variables, constants, functional expressions (can be arguments to predicates)
- Sentences:
  - atomic sentences (predicate expressions),
  - complex sentences (atomic sentences connected by Booleans),
  - quantified sentences

Examples of Terms: Constants, Variables and Functions

- Constants
  - Alan, Sam, R225, R216
- Variables
  - PersonX, PersonY, RoomS, RoomT
- Functions
  - father_of(PersonX)
  - product_of(Number1,Number2)

Examples of Predicates and Quantifiers

- Predicates
  - In(Octavian, Room4)
  - part of(Room4, UC)
  - fatherOf(PersonX,PersonY)

- Quantifiers
  - All dogs are mammals.
  - Some birds can't fly.
  - 3 birds can't fly.

Semantics of Predicate Logic

- A term is a reference to an object
  - constants
  - variables
  - functional expressions
- Sentences make claims about objects
  - Well-formed formulas, (wffs)

Semantics, part 2

- object constants refer to individuals
- there is a correspondence between
  - functions, which return values
  - predicates, which are true or false

Function: father_of(Mary) = Bill

Predicate: father_of(Mary, Bill)

Semantics, part 3

- Referring to individuals
  - Jackie
  - son-of(Jackie), Sam
- Referring to states of the world
  - person(Jackie), female(Jackie)
  - mother(Sam, Jackie)

Encoding Facts, (cont.)

pass(John, courses,40) **=>** graduate(John)

cavity(molar) **=>** x-ray_shadow(molar)

leak(pipe, kitchen) /\ full(pipe,water) **=>**
location(water, kitchen_floor)

KB Design Choices

Design choice:	Implication of choice:
• red(block1)	?????
• color(block1, red)	nice(red)
• val(color,block1,red)	property(color)

Combining Logical Symbols

- Terms: logical expressions referring to objects
  - first([a,b,c]), sq_root(9), sq_root(n), tail([a,b,c])
- Atomic Sentences:
  - loves(John,Mary), brother_of(John,Ted)
- Complex Sentences:
  - loves(John,Mary) $\Rightarrow$ brother_of(John,Ted) $\wedge$ teases(Ted, John)

Quantifiers

- Universal Quantification

  All cats are mammals.

$$\forall x, Cat(x) \Rightarrow Mammal(x)$$

– For all x, such that x is a cat, x is a mammal

$$Cat(Garfield) \Rightarrow Mammal(Garfield)$$

Quantifiers

- Existential Quantifiers

A cat has an owner named John.

OR

There exists an entity such that that entity is a cat and has an owner named John.

$$\exists x, Owner(x, John), Cat(x)$$
$$Owner(Garfield, John), Cat(Garfield)$$
$$Owner(Rueben, John), Cat(Rueben)$$

Nested Quantifiers

$$\forall x, \exists y,\ Cat(x) \Rightarrow Owner(x, y)$$

$$Cat(Garfield) \Rightarrow Owner(Garfield, John)$$

$$Cat(Rueben) \Rightarrow Owner(Rueben, John)$$

$$\forall x, \forall y,\ owner(x, y) \Rightarrow loves(x, y)$$

$$owner(Rueben, John) \Rightarrow loves(Rueben, John)$$

$$owner(Garfield, John) \Rightarrow loves(Garfield, John)$$

Unique variable names!

First–Order Logic Examples

$\forall person\ ItIsRaining() \rightarrow IsWet(person)$

- Objects: *john, chair23, mother-Of(john)*
- Relations: *isWet(john)*, *isSittingOn(john,chair23)*
- Complex sentences:
  - Boolean connectives: *person(john)* → *likes(john, chocolate)*
  - Quantifiers and variables:

    (∀*person)likes(person,chocolate)*

    (∃*person)* ¬*eat(person,chocolate)*

More First-Order Logic Examples

John loves Mary.

All crows are black.

Dolphins are mammals that live in the water.

Mary likes the color of one of John’s ties

The Power of Expressivity

- Indirect knowledge:
  Tall(MotherOf(john))
- Counterfactuals: ¬*Tall(john)*
- Partial knowledge (disjunction):
  IsSisterOf(b,a) ∨ *IsSisterOf(c,a)*
- Partial knowledge (indefiniteness):
  ∃*xIsSisterOf(x,a)*

R& N, Chapter 15