"UBC-ALM: Combining k-NN with SVD for WSD"

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UBC-ALM: Combining k-NN and SVD for WSD

This 2007 paper by Agirre and Lacalle looks at representing senses of a word with feature vectors and combining two mathematical methods to do WSD using those vectors.

Outline

Feature Vectors

The idea is to represent a word with a vector (a list of numbers).

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One of the simplest ways to do this is what is known as the bag-of-words approach.

Feature Vectors

If you have the sentence:

"The cat ran."

Then the bag-of-words representation for the words in this sentence is to have a vector where the only non-zero entries are:

the $=$ 1, cat $=$1, ran $=$1

basically, just having a value of 1 for every position in the vector corresponding to a word that appears in the sentence.

Feature Vectors

To make this into a vector, all we do is associate each entry in the vector with a word.

For the "The cat ran." example, this vector would look like

$$\begin{pmatrix} | \\ A_1 \\ | \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

Again, where the 1's are in the slots for "the", "cat", and "ran", and all the others are 0.

Feature Vectors

When we collect a set of such vectors into a single object like so

$$\begin{pmatrix} | & | \\ A_1 & A_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 1 & \vdots \\ 0 & 1 \\ \vdots & 0 \\ 1 & \vdots \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \end{pmatrix}$$

Feature Vectors

We have what is called a matrix.

$$\begin{pmatrix} | & | \\ A_1 & A_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 1 & \vdots \\ 0 & 1 \\ \vdots & 0 \\ 1 & \vdots \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \end{pmatrix}$$

Feature Vectors

I'm only using two vectors for illustration, you can include any number and still have a matrix.

$$\begin{pmatrix} | & | & & | \\ A_1 & A_2 & \dots & A_n \\ | & | & & | \end{pmatrix} = \begin{pmatrix} 1 & \vdots & & \vdots \\ 0 & 1 & & 0 \\ \vdots & 0 & & 1 \\ 1 & \vdots & \dots & \vdots \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 0 & & 1 \end{pmatrix}$$

Feature Vectors

This leads us into SVD.

SVD

SVD stands for Singular Value Decomposition.

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In mathy terms, it's where you take a matrix, A, and break it up into the product of three special matrices.

SVD of A

$$A = U\Sigma V^T$$

U contains the left singular vectors of the matrix A, Σ contains its singular values, and V contains its right singular vectors.

SVD

This is what it looks like for a 3 by 3 matrix.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$

SVD

The terms in the middle matrix, the σ's, are what are called the "singular values".

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$

SVD

When we delete the singular values the farthest along the diagonal, we still have a good approximation to our original matrix A

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\approx$$

$$\begin{pmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \mathbf{0} \end{pmatrix} \begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$

SVD

This implies we can approximate the three column vectors of the A matrix by only using the first two u vectors.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\approx$$

$$\begin{pmatrix} | & | & | \\ \mathbf{u_1} & \mathbf{u_2} & u_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} \boldsymbol{\sigma_1} & 0 & 0 \\ 0 & \boldsymbol{\sigma_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$

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You have to know how matrix multiplication works to see this, but the idea of using SVD to represent more vectors with fewer is the important thing.

SVD

So what do these approximations do?

SVD

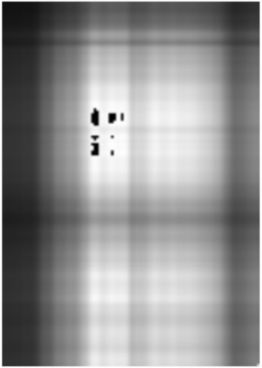
Let's ask Big Bird!

SVD

Grayscale images are represented with numbers that determine by how bright to make each pixel, meaning this 104 by 138 pixel image is actually represented by a matrix.

SVD

Using 1 Singular Value



SVD

Using 5 Singular Values

SVD

Using 40 Singular Values

SVD

Using All 104 Singular Values

SVD

As you can see, we can get a pretty good approximation to what all the vectors comprising the image are doing, using only a fraction of the singular vectors.

SVD

Moreover, with regards to WSD it isn't that we want this

Using 40 Singular Values

SVD

We actually want this

Using 5 Singular Values

SVD

The reason is that while using fewer singular values blurs the image, this is tantamount to adding in unseen word counts with respect to our vector description of words.

Using 5 Singular Values

SVD

It's a double whammy! Not only do we have fewer vectors to deal with, but we also get guesstimates to word counts we haven't seen in collecting data, but suspect are relevant.

Using 5 Singular Values

SVD

So instead of using simple bag-of-words vectors, we instead use these singular vectors to describe senses of a word as they help with sparsity of data issues.

k-NN

The last piece of the puzzle is to use k-NN (k-Nearest Neighbors) to disambiguate words.

k-NN

Before understanding how it works, one needs to realize that vectors have a geometric interpretation as points in space.

k-NN

For example, take the vector

$$\begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

k-NN

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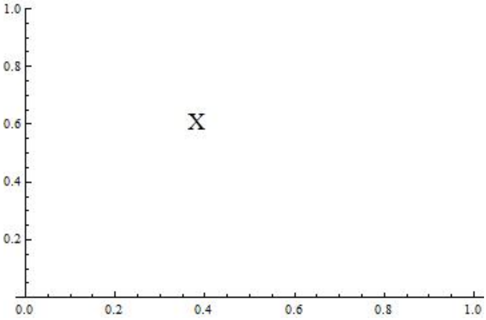
$$\begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

This can be represented graphically as

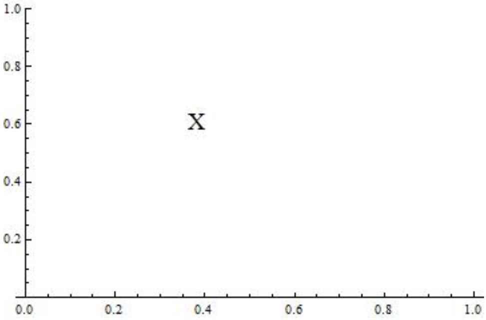
k-NN

Graphically Representing (0.4,0.6)



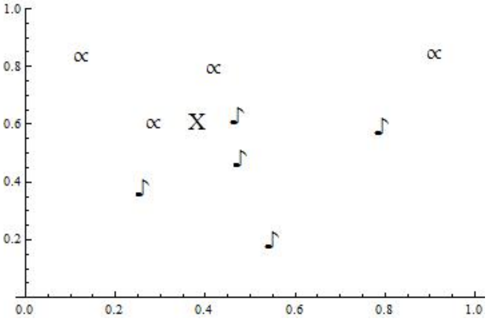
k-NN

For the sake of illustration, say that this "X" represents the word "bass" and we are trying to find the sense for it.



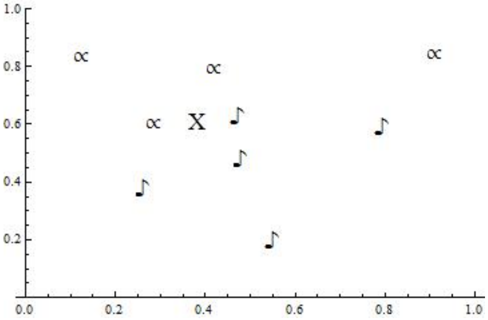
k-NN

Now add in the points that have been tagged in training our WSD classifier.



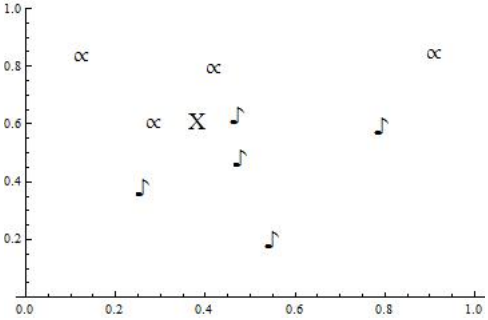
k-NN

Let $\propto$ represent the "fish" version and the eighth-notes represent the music version for feature vectors from a hand-tagged corpus.



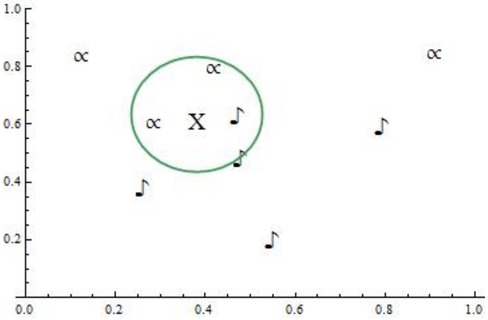
k-NN

The way k-NN works is to assign the label to the target word that is shared the most by its k nearest hand-tagged neighbors.



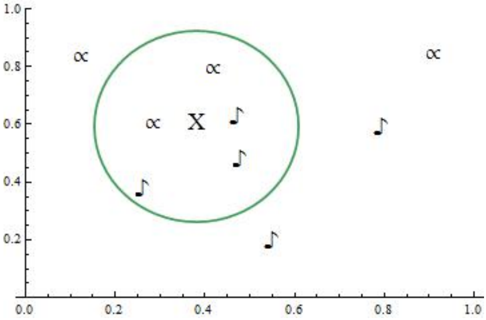
k-NN

For example, if $k = 3$ then in this example we would say that the target word has the fish sense .



k-NN

If $k = 5$, then the music sense.



Paper's Results

In the paper, all of these approaches had some extra bells and whistles thrown on them (e.g. the feature vectors weren't just bag-of-words, and they used a weighted k-NN), but nothing really worth getting into.

If you've understood what I've said so far, then you understand what they were fundamentally doing in the paper.

Paper's Results

They compared their WSD classifier to the others entered in the SemEval-2007 on the Lexical Sample and All-Words tasks, and overall did pretty well:

Task	Method	Rank	rec.	prec.
LS	Best	1	0.887	0.887
LS	UBC-ALM	2	0.869	0.869
LS	Baseline	-	0.780	0.780
AW	Best	1	0.591	0.591
AW	k-NN combination	5	0.544	0.544
AW	Baseline	-	0.514	0.514

Questions?

Thank you for your time.