Finite state morphology and phonology

Natural Language Processing
LING/CSCI 5832

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Jan 20 2014

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FSMs for practical NLP tasks

(1) How FSMs are used in modeling sound systems (phonology)

(2) For modeling word-formation

(3) Derivative products of the above (spell checkers, lemmatizers, grammar checkers, components of larger systems)

Plan

(1) Recap finite automata and transducers + basic algorithms

(2) Look at an extended calculus for manipulating FSMs (automata + transducers) suitable for NLP

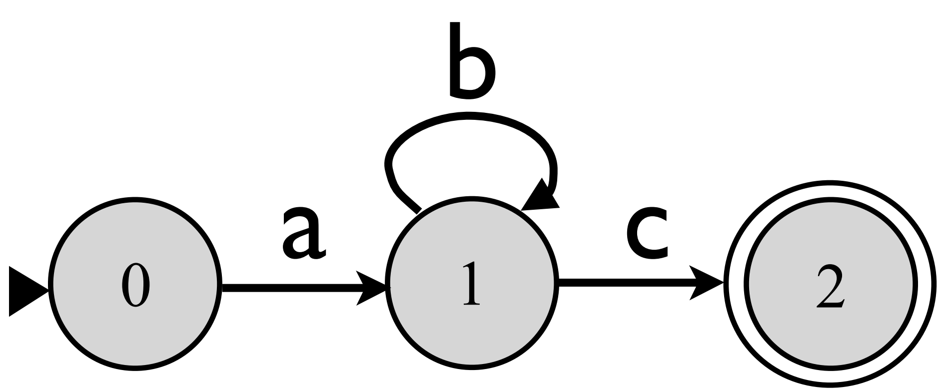
(3) See how these are used in natural language applications

Recap: anatomy of a FSA

Regular expression

L = a b* c

Graph representation



Formal definition

$Q = \{0,1,2\}$ (set of states)

$\Sigma = \{a,b,c\}$ (alphabet)

$q_0 = 0$ (initial state)

$F = \{2\}$ (set of final states)

$\delta(0,a) = 1,\ \delta(1,b) = 1,\ \delta(1,c) = 2$
(transition function)

Recap: anatomy of a FSA

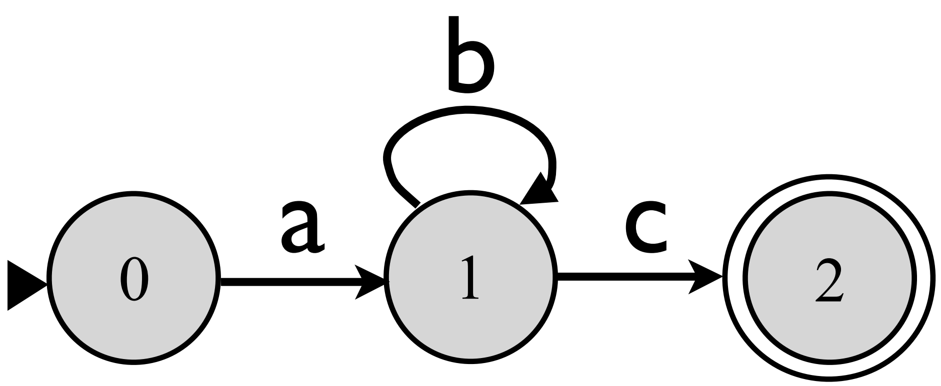
Regular expression

L = a b* c

Interpretation

- An FSA defines a **set of strings**
- In this case L={ac,abc,abbc,...}
- These sets are the **regular sets**

Graph representation



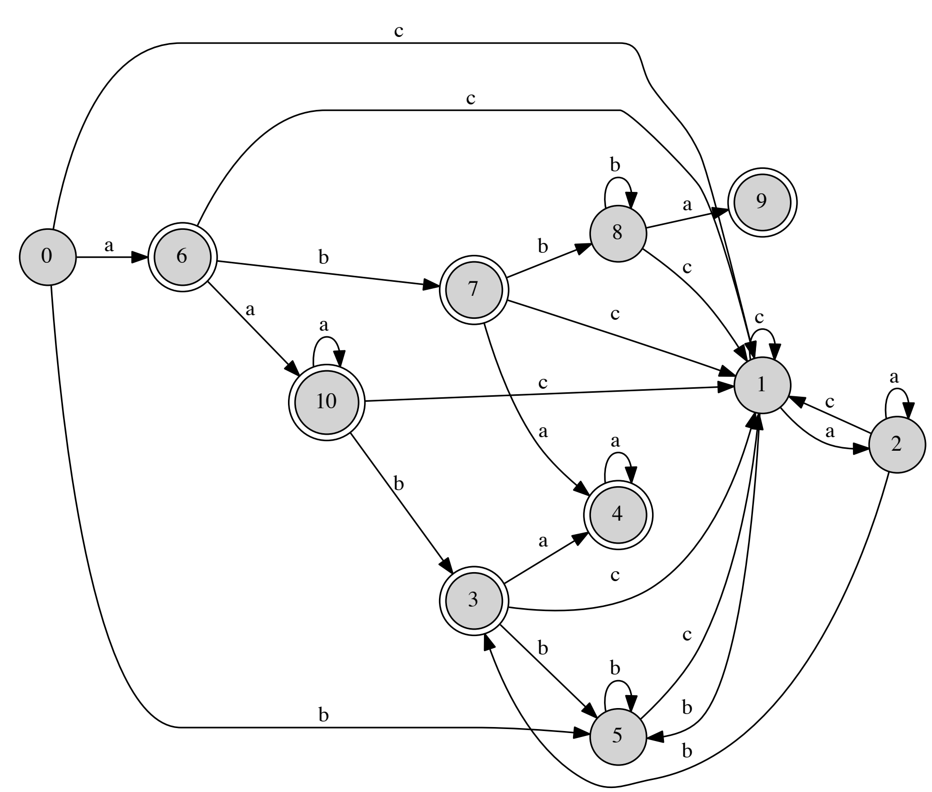
Recap: Kleene's Theorem

A language is **regular** iff it is accepted by some FA

Proof is constructive: can convert between representations

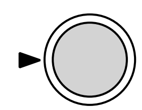
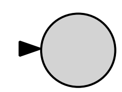
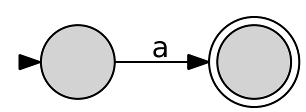
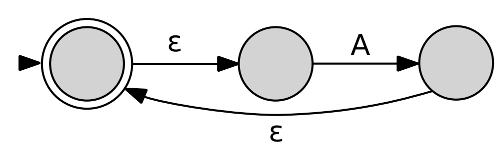
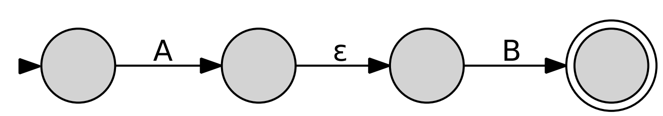
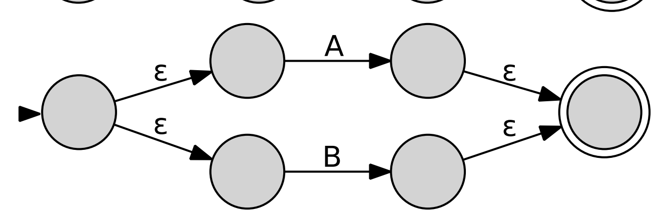
$$(a|b^*\ c)^*\ a\ b\ a^*\ |\ (a\ b^*\ a\ |\ a\ a^*)$$

⇕



Recap: Kleene’s Theorem

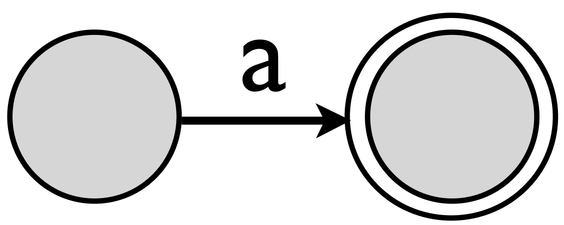
Kleene’s Theorem: regexp → FA

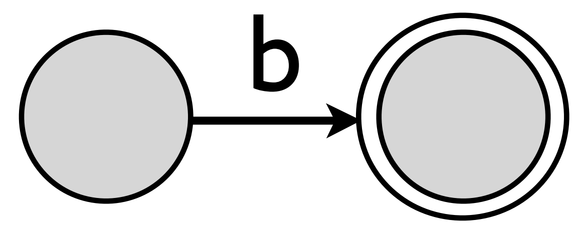
Expression	Definition	FSM construction
ϵ	The empty string	
$\emptyset$	The empty language	
a	A single symbol	
A^*	Kleene star of a language	
AB	Concatenation of two languages	
$A \mid B$	Union of two languages	

FA → regexp done with “state elimination algorithm” (easier, but let’s skip it)

The Thompson construction

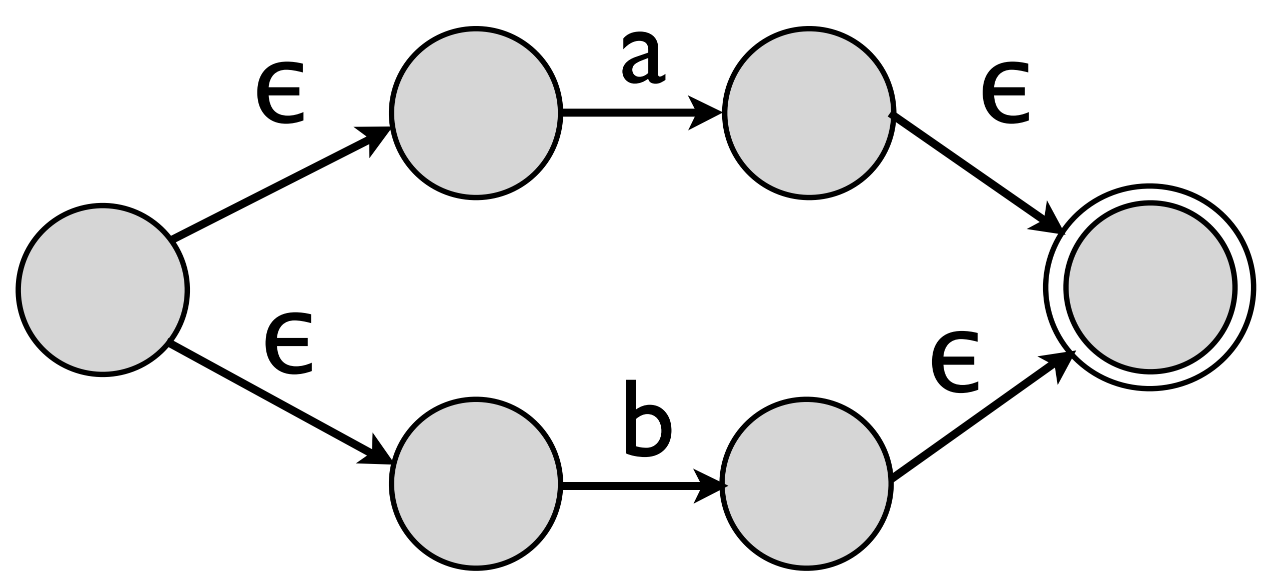
(a|b)*





The Thompson construction

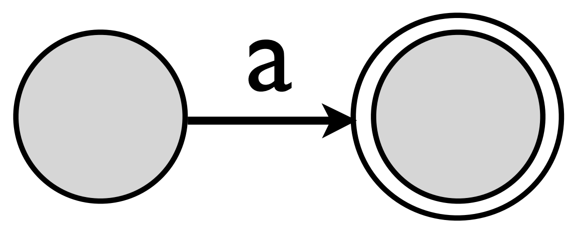
(a|b)*

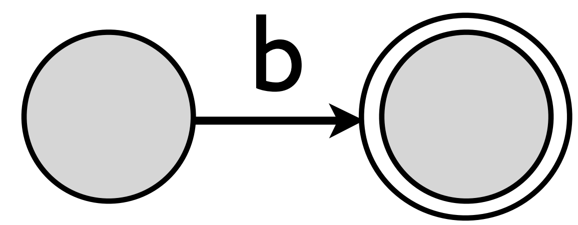


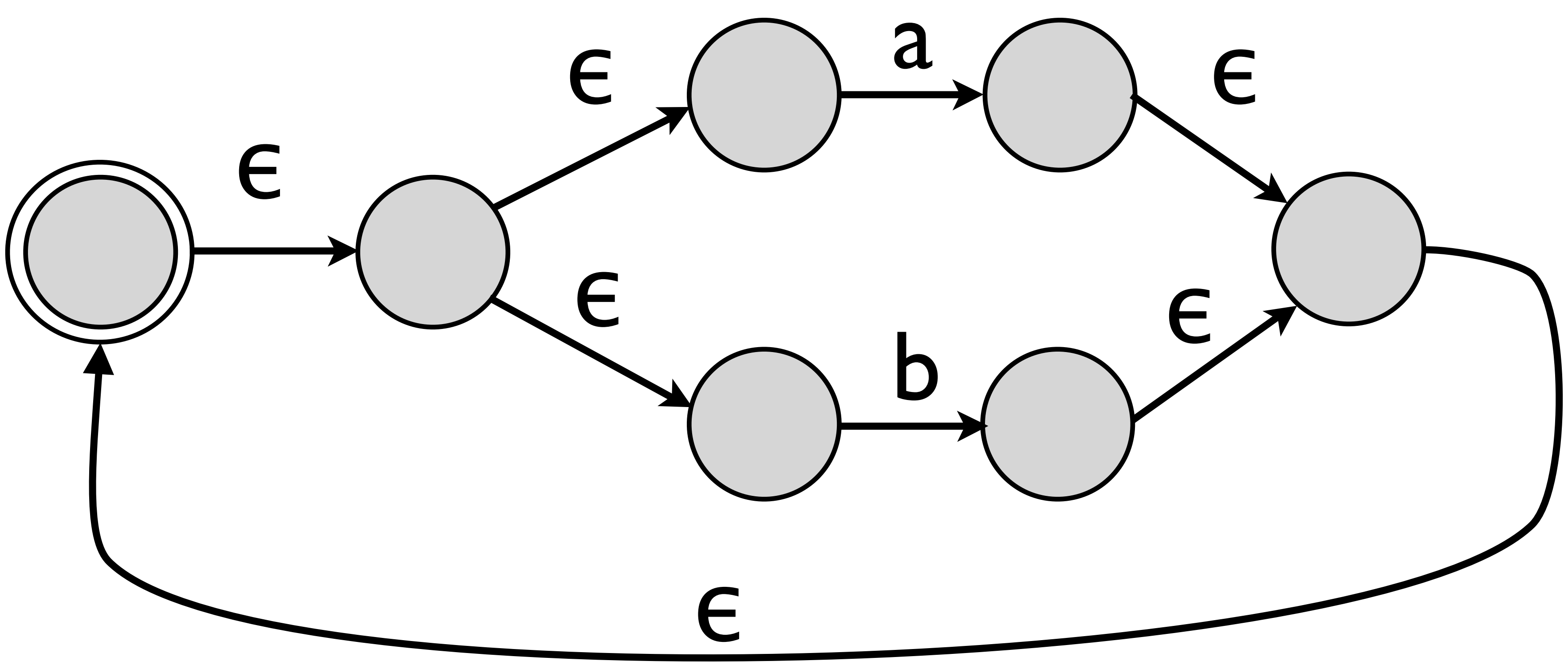
(a|b)

The Thompson construction

(a|b)*

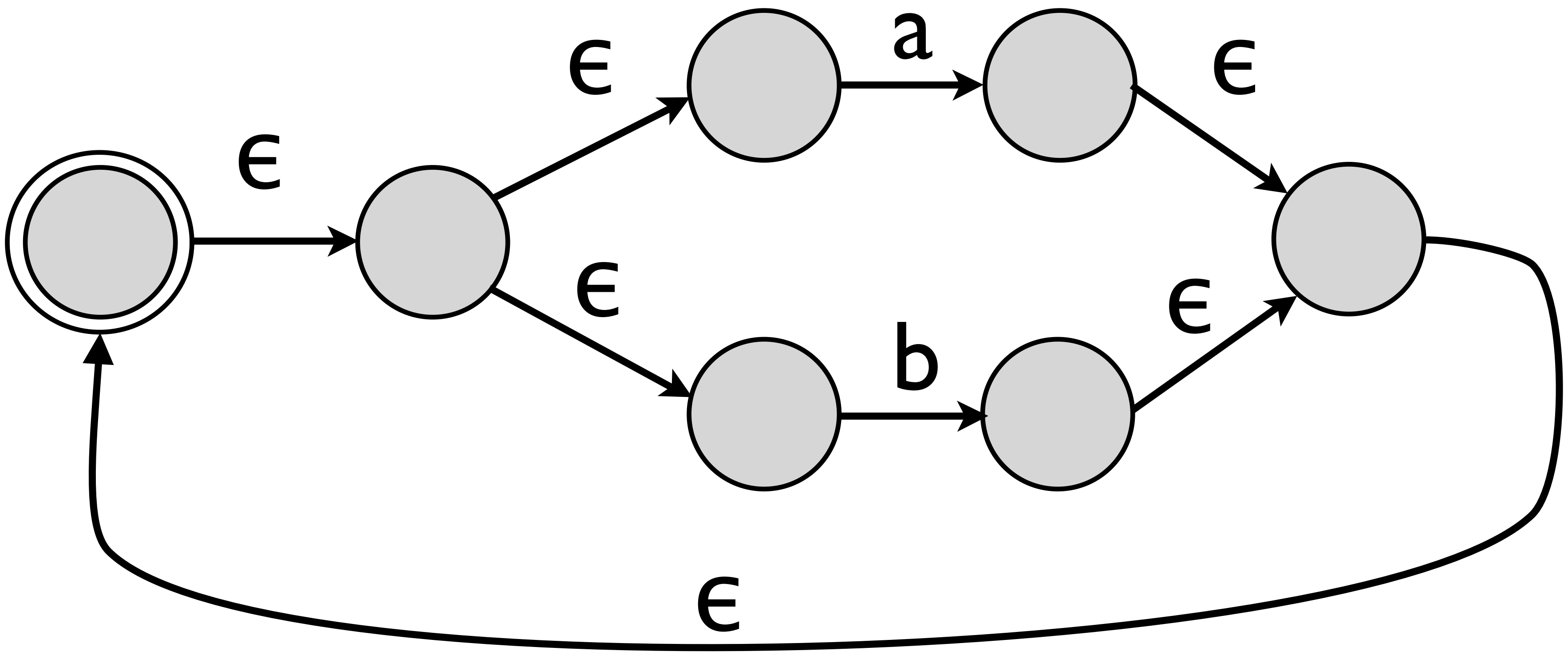




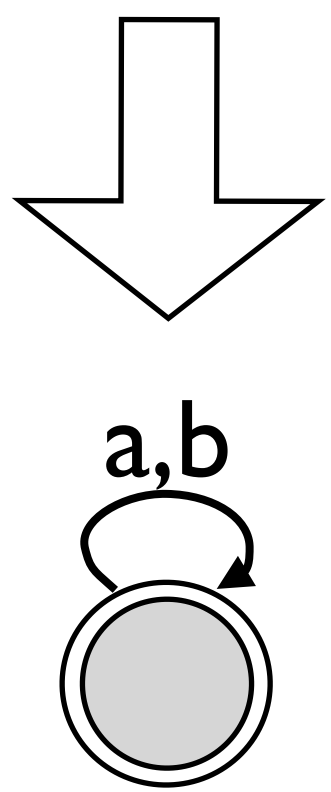


(a|b)*

The Thompson construction



(a|b)*



determinization &
minimization algorithm

Recap: Kleene's Theorem

- Kleene's Theorem only uses one Boolean operation on sets, union

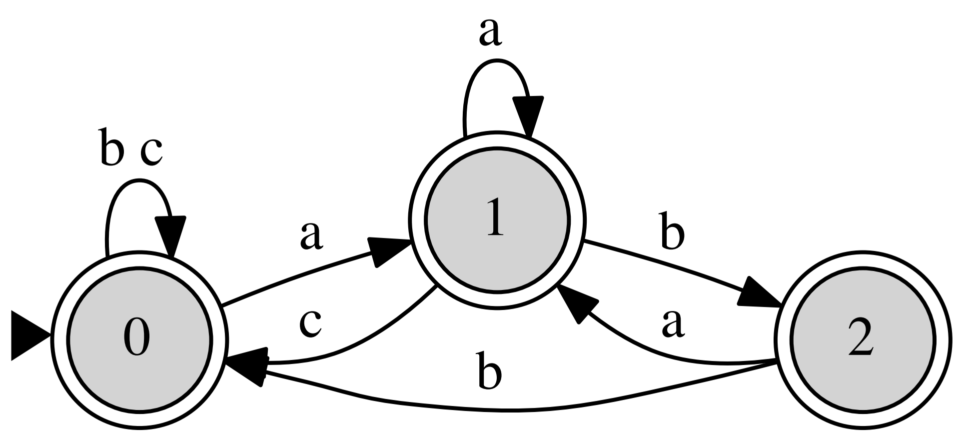
- But FSA are closed under other set operations: complement, intersection, set subtraction

- It's difficult to appreciate the power of finite-state models without a richer calculus...

- In fact, the most fruitful approach is to forget about states and transitions and tapes and reason in terms of sets and relations

Reasoning about automata

Automaton

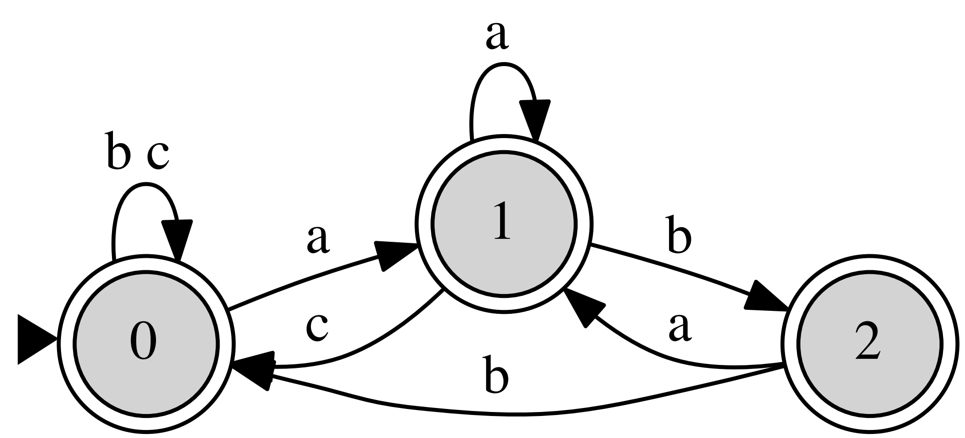


$$\Sigma = \{a,b,c\}$$

What language does the FSA represent?

Reasoning about automata

Automaton



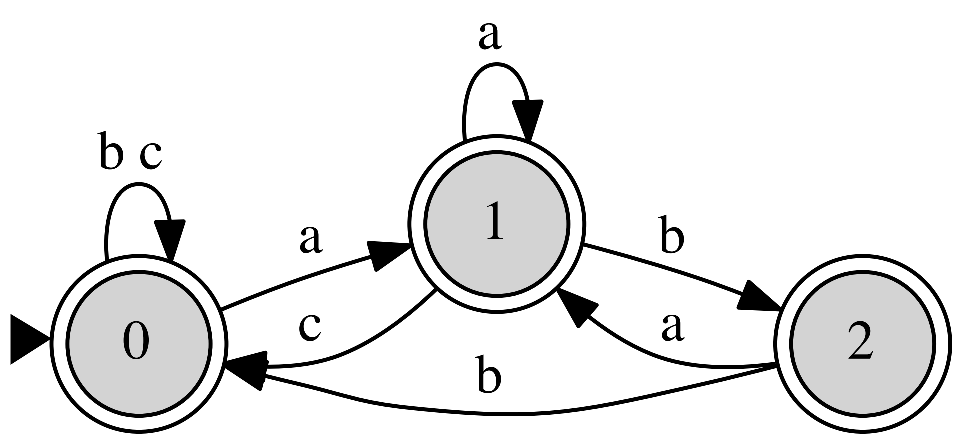
$\Sigma = \{a,b,c\}$

Equivalent regular expression with {|,., *}

(b|c|aa*c)*aa*b(aa*b|(b|aa*c)(b|c|aa*c)*aa*b)*|(b|c)* a((a|ba)|(c|bb)(b|c)*a)*|(b|c|a(a|ba)*(c|bb))*

Reasoning about automata

Automaton



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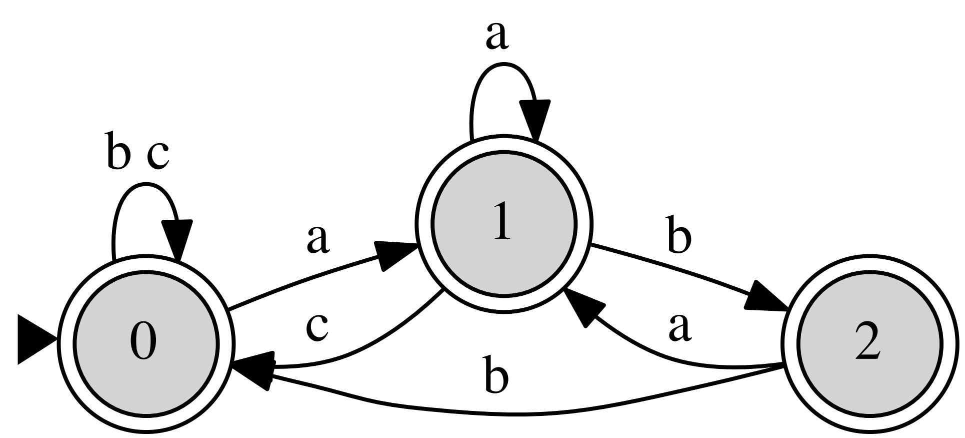
Equivalent regular expression with {.,¬,*}

$\neg(\Sigma^* abc \Sigma^*)$

Reasoning about automata

Automaton



$\Sigma = \{a,b,c\}$

Equivalent regular expression with {|,·, *}

(b|c|aa*c)*aa*b(aa*b|(b|aa*c)(b|c|aa*c)*aa*b)*|(b|c)* a((a|ba)|(c|bb)(b|c)*a)*|(b|c|a(a|ba)*(c|bb))*

Equivalent regular expression with {|,·,¬}

¬(Σ*abcΣ*)

not "contains abc"

Reasoning about automata

From “Regular models of phonological rule systems”

The common data structures that our programs manipulate are clearly states, transitions, labels, and label pairs—the building blocks of finite automata and transducers. But many of our initial mistakes and failures arose from attempting also to think in terms of these objects. The automata required to implement even the simplest examples are large and involve considerable subtlety for their construction. To view them from the perspective of states and transitions is much like predicting weather patterns by studying the movements of atoms and molecules or inverting a matrix with a Turing machine. The only hope of success in this domain lies in developing an appropriate set of high-level algebraic operators for reasoning about languages and relations and for justifying a corresponding set of operators and automata for computation.

(Kaplan and Kay, 1994, p.376)

Toward “high-level” algebraic operators

- Add Booleans to regular expression calculus: at least complement (¬), intersection (∩), set subtraction (-))

- Add “useful” operators/shortcuts, e.g.
  - contains(X) = (Σ* X Σ*)

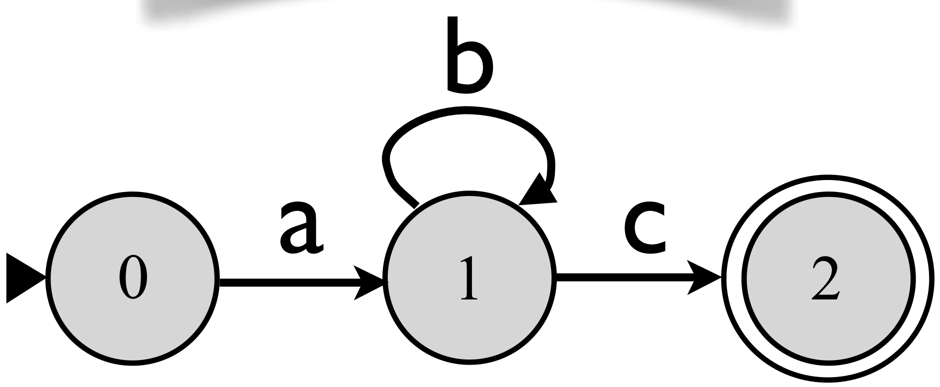
- Example: the language that fulfills the constraint: “i before e except after c”

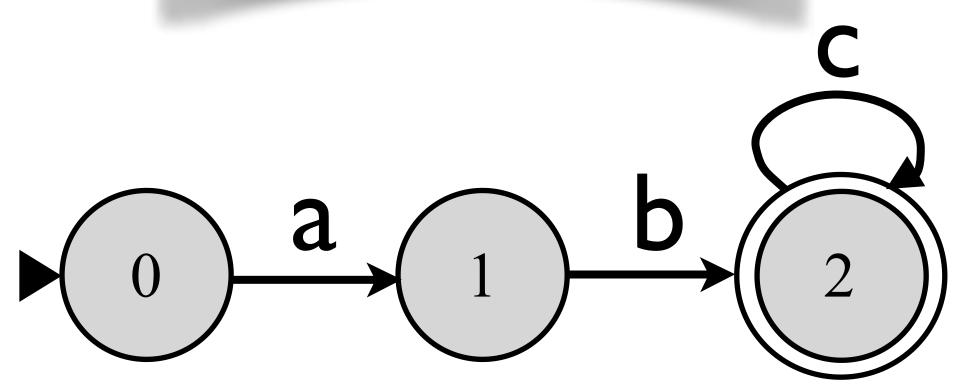
  ¬contains(cie) & ¬(¬(Σ*c)ei)

The product construction

$L_1 = a\,b^*\,c$

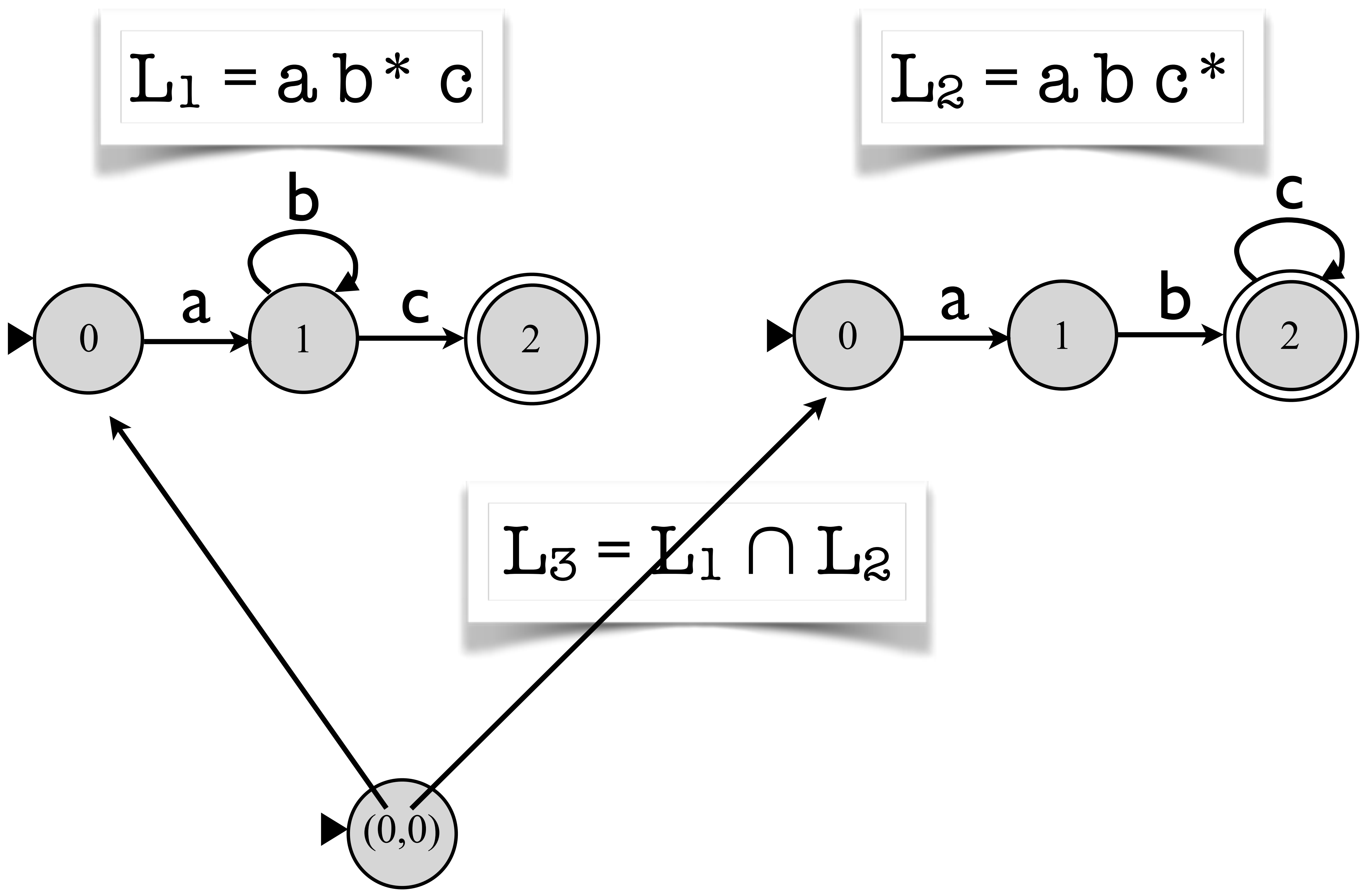
$L_2 = a\,b\,c^*$



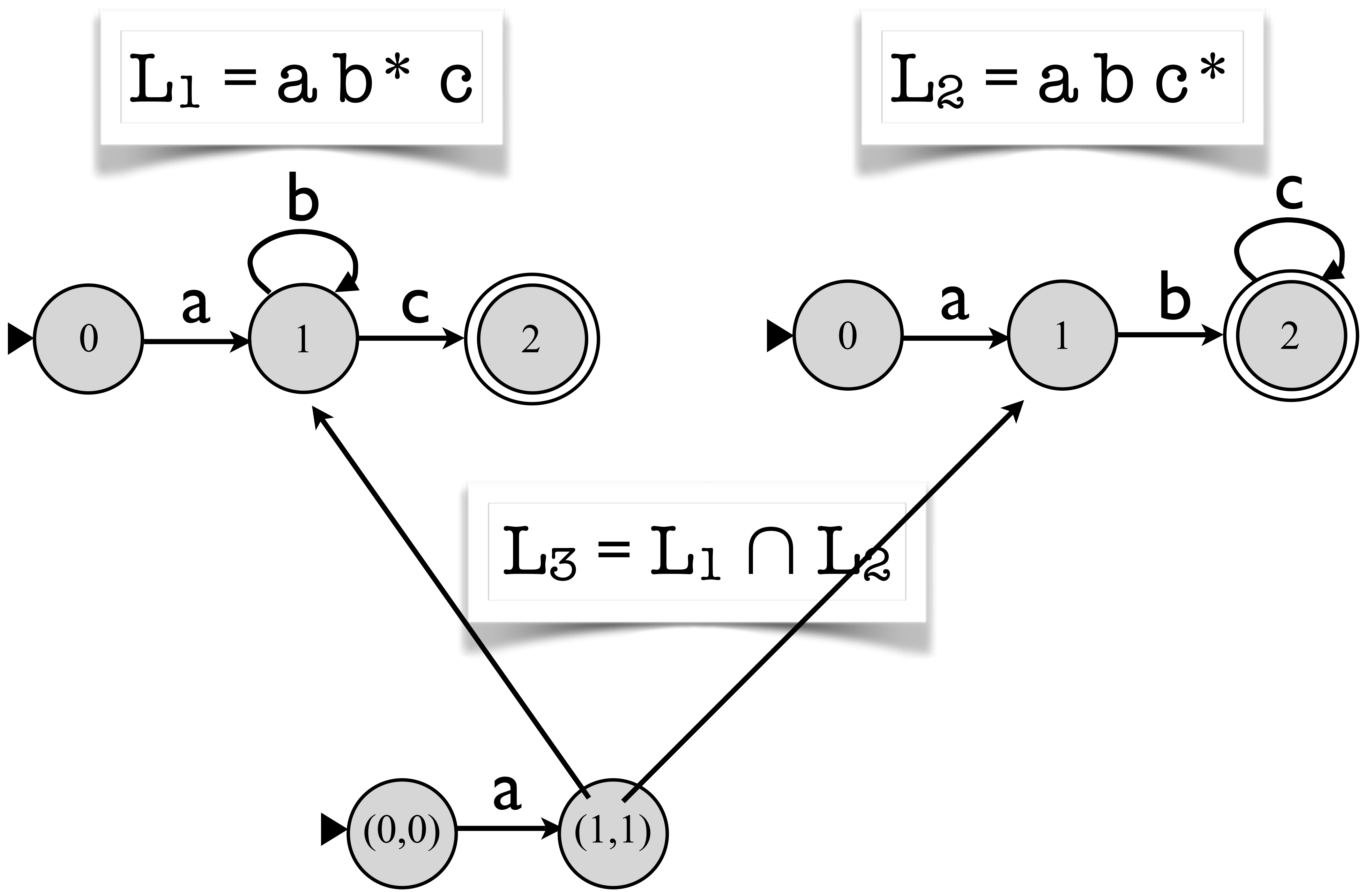


$L_3 = L_1 \cap L_2$

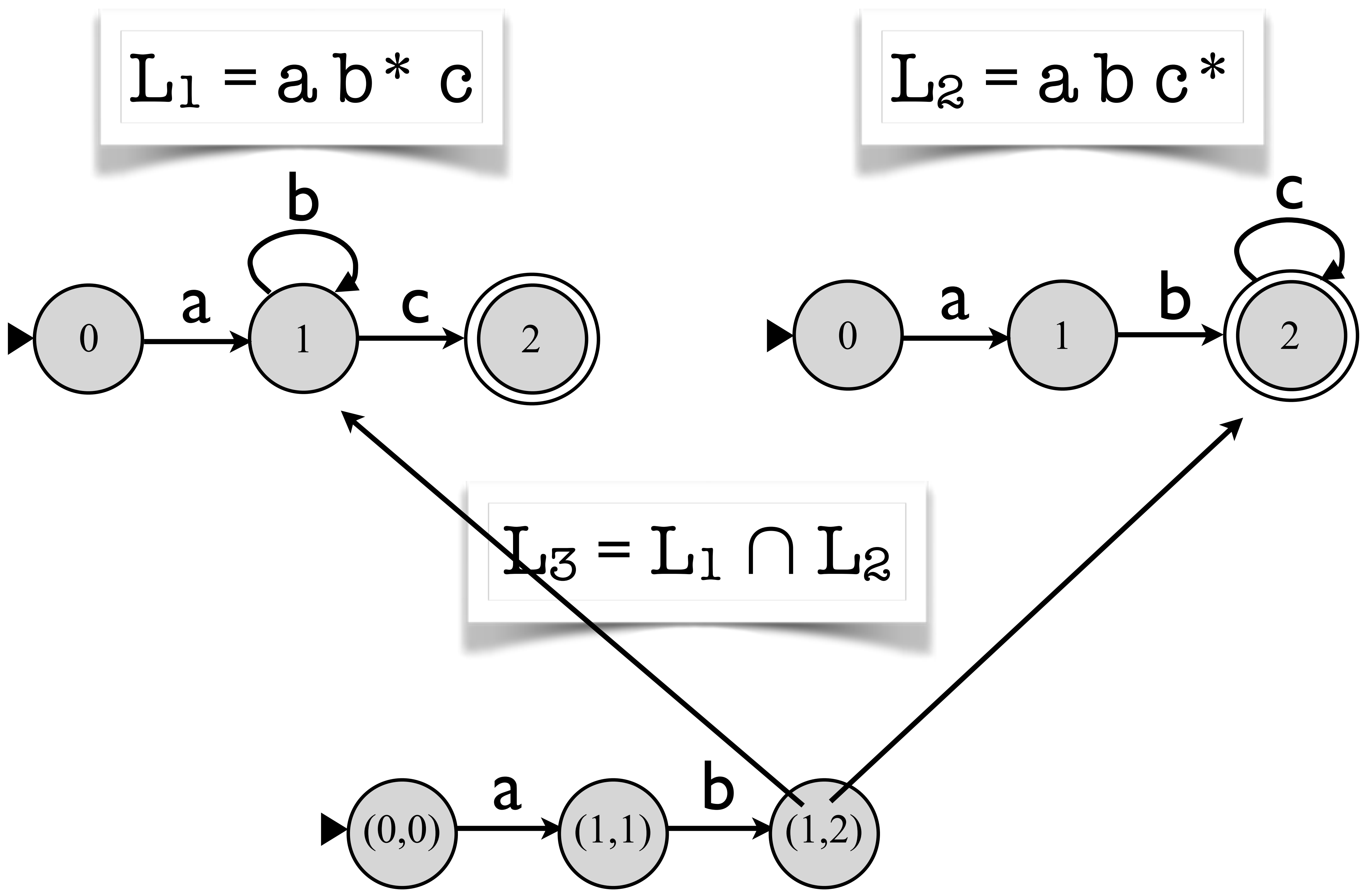
The product construction



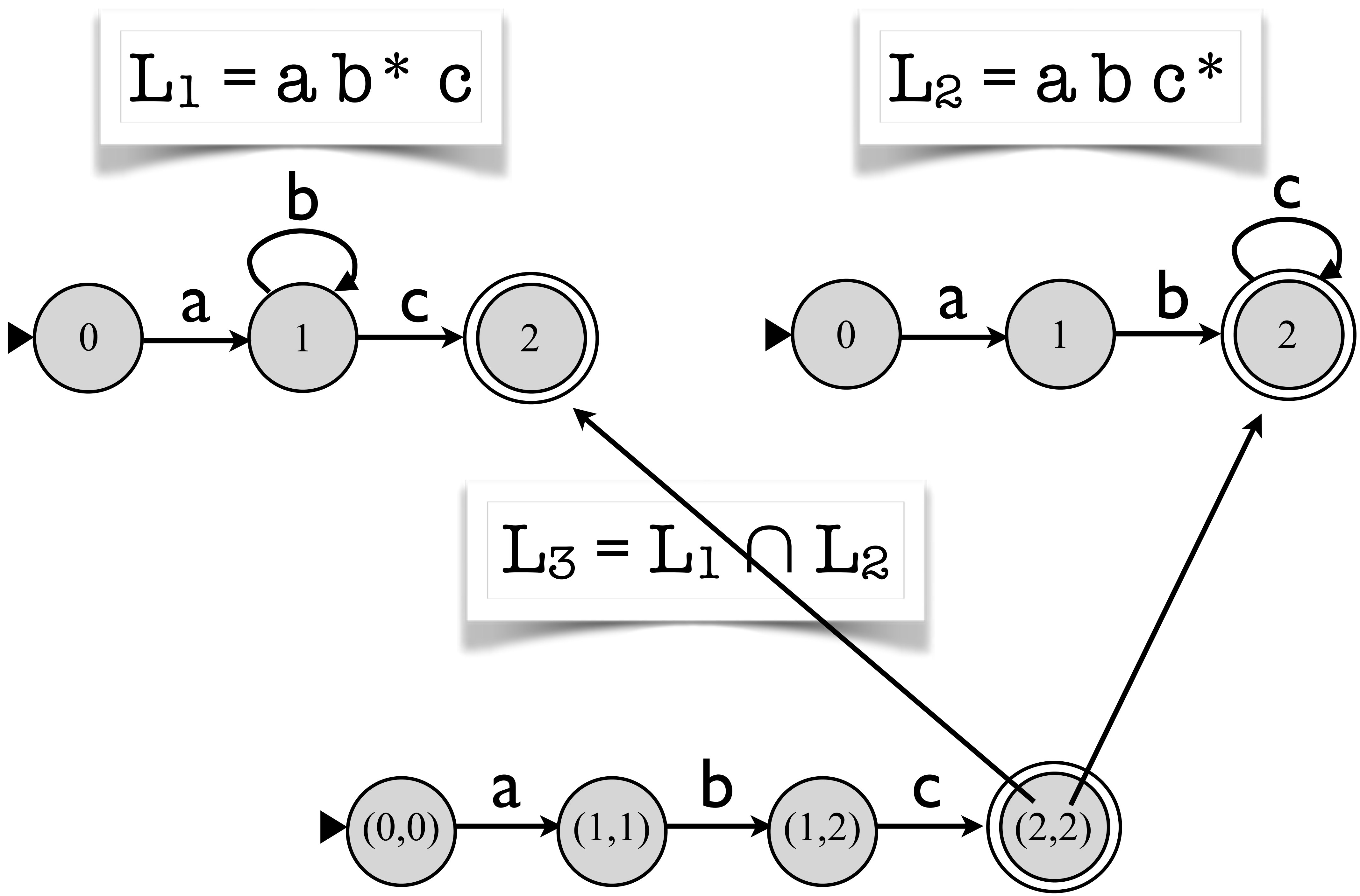
The product construction



The product construction



The product construction



The product construction

Algorithm 3.2: PRODUCTCONSTRUCTION

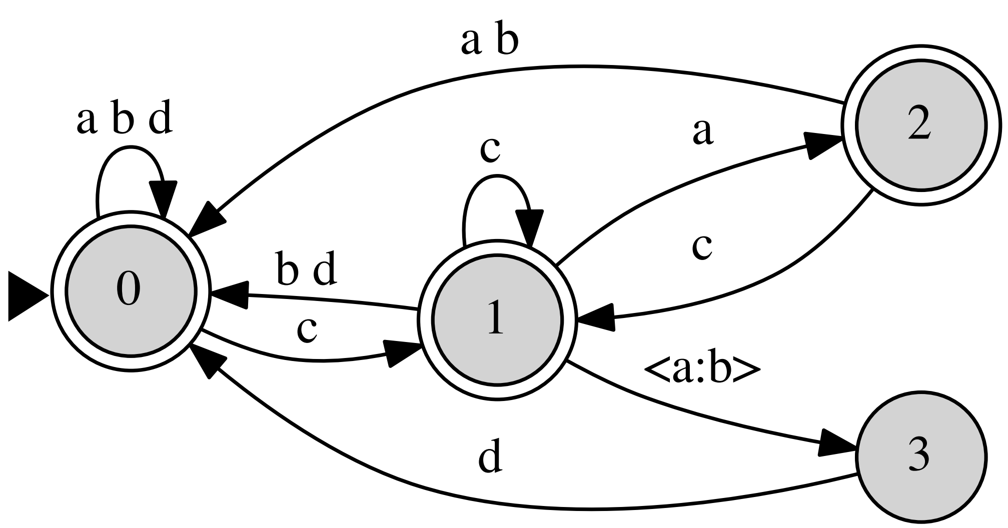
Input: $FSM_1 = (Q_1, \Sigma, \delta_1, s_0, F_1), FSM_2 = (Q_2, \Sigma, \delta_2, t_0, F_2)$, OP $\in \{\cup, \cap, -\}$
Output: $FSM_3 = (Q_3, \Sigma, \delta_3, u_0, F_3)$

begin
    Agenda $\leftarrow (s_0, t_0)$
    $Q_3 \leftarrow (s_0, t_0)$
    $u_0 \leftarrow (s_0, t_0)$
    index (s_0, t_0)
    while *Agenda* $\neq \emptyset$ **do**
        Choose a state pair (p, q) from Agenda
        foreach *pair of transitions* $\delta_1(p, x, p')$ $\delta_2(q, x, q')$ **do**
            Add $\delta_3((p, q), x, (p', q'))$
            if *(p',q') is not indexed* **then**
                Index (p', q') and add to Agenda and Q_3
            end
        end
    end
    foreach *State s in Q_3* = (p, q) **do**
        Add s to F_3 iff $p \in F_1$ OP $q \in F_2$
    end
end

Finite state transducers

Recap: anatomy of an FST

Graph representation



Formal definition

$Q = \{0,1,2,3\}$ (set of states)

$\Sigma = \{a,b,c,d\}$ (alphabet)

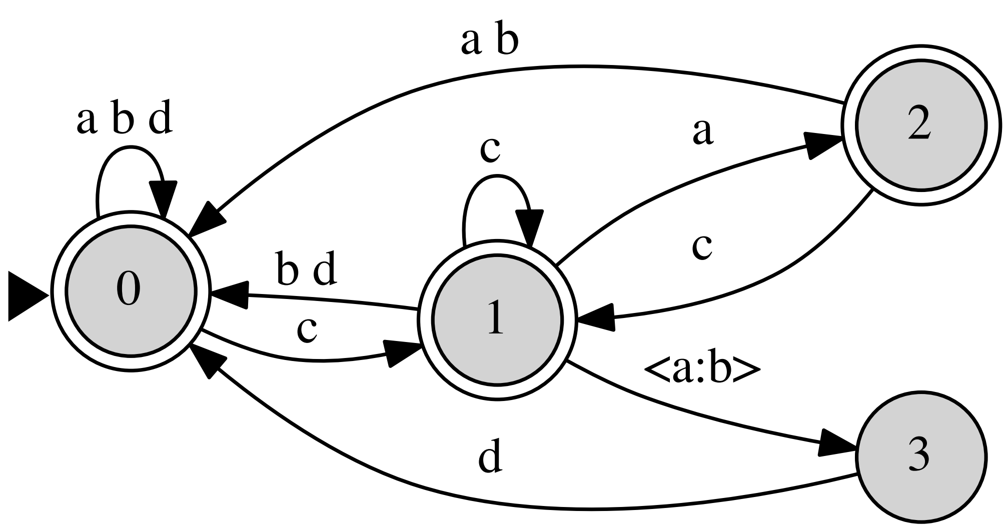
$q_0 = 0$ (initial state)

$F = \{0,1,2\}$ (set of final states)

δ (transition function)

Recap: anatomy of an FST

Graph representation



Interpretation

- An FST defines a **set of string pairs (a relation)**
- In this case T={(a,a),(b,b),(c,c),(cad,cdb),...}
- These sets are the **regular relations**
- Trivially bidirectional devices

Algebraic operations on transducers

T U (concatenation)

T | U (union)

T* (Kleene closure)

rev(T) (reversal)

L_1 x L_2 (cross-product)

T o U (composition)

Algebraic operations on transducers

T U (concatenation)

T | U (union)

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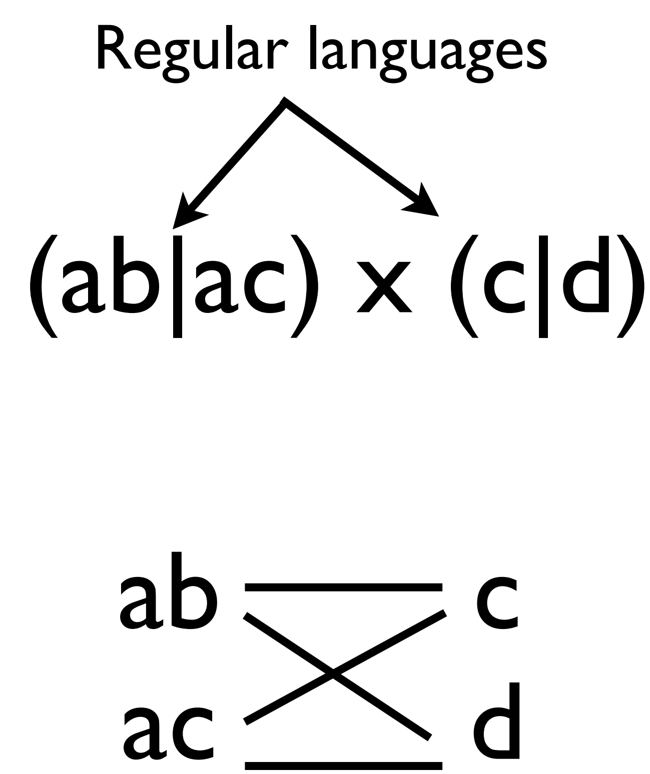
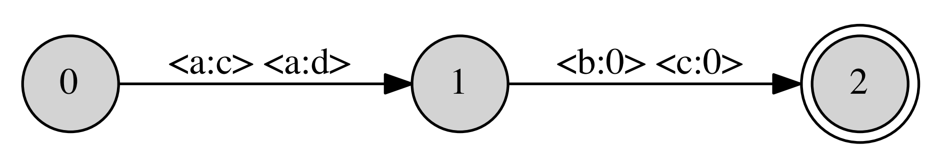
rev(T) (reversal)

L_1 x L_2 (cross-product)

T o U (composition)

Cross-product





Algebraic operations on transducers

T U (concatenation)

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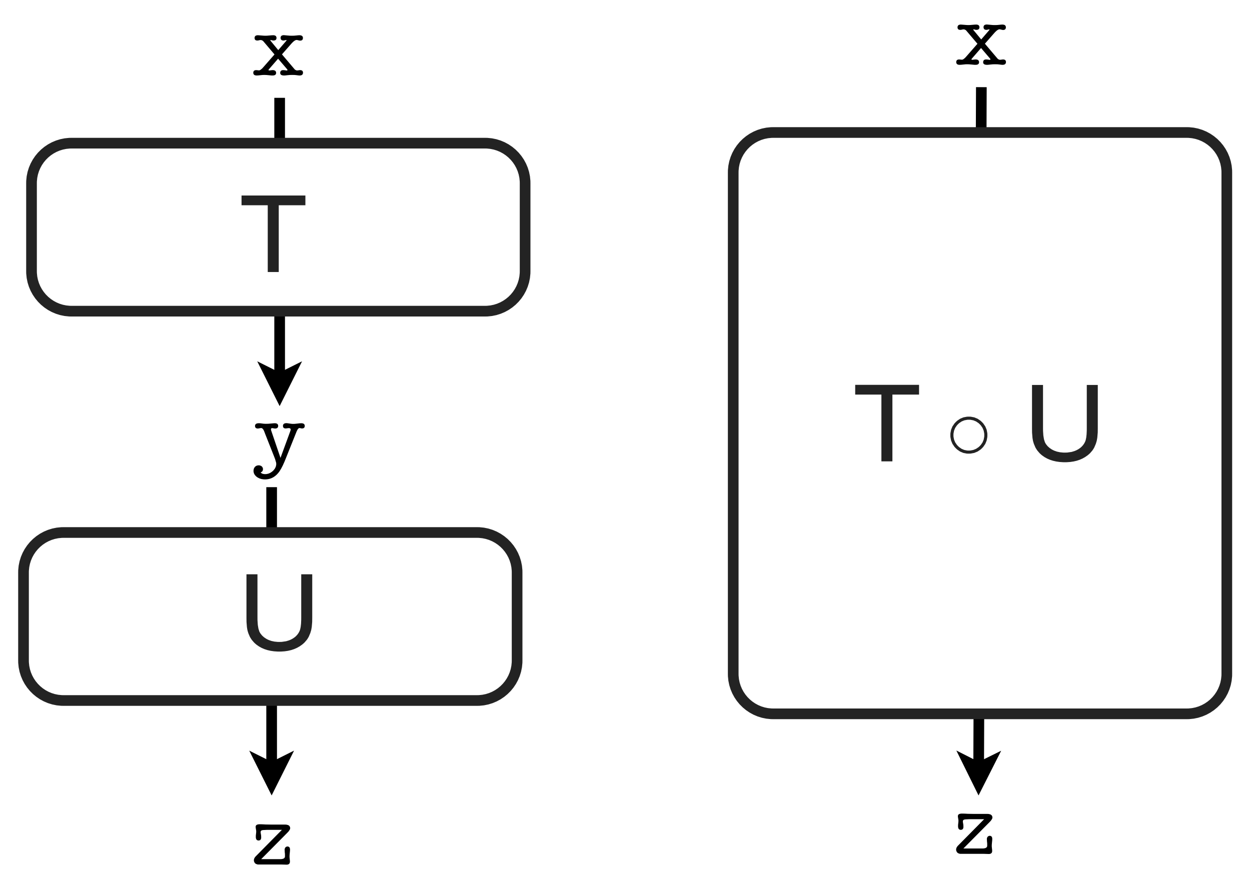
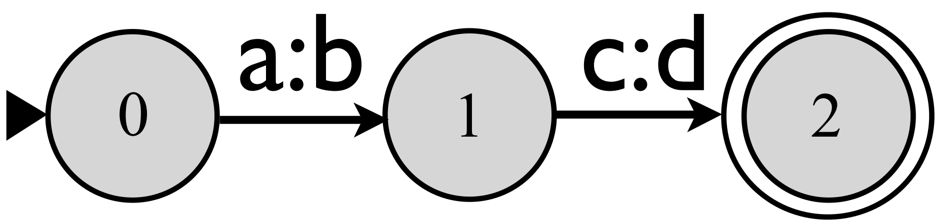
T o U (composition)

Composition

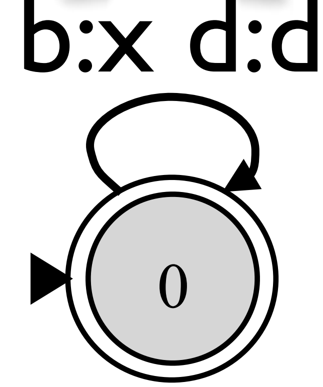


Composition: product construction

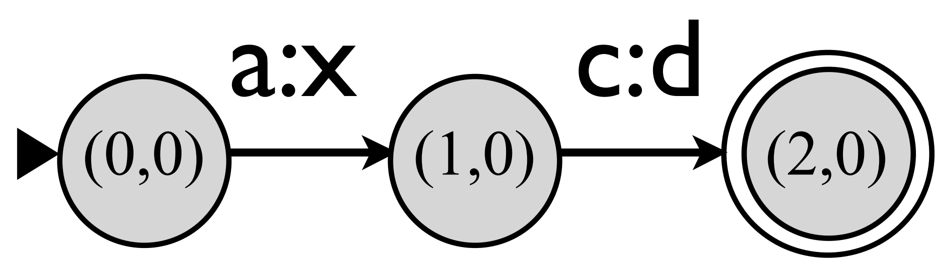
T_1



T_2



$T_3 = T_1 \circ T_2$



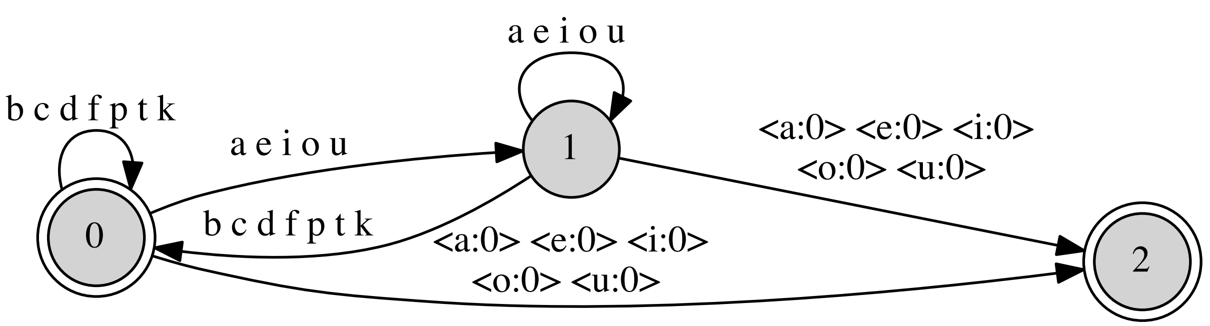
String rewriting operators

A →B / C _ D

"Rewrite strings A as B when occuring between C and D"

Example: (a|e|i|o|u) → 0 / _ #

delete vowels at the ends of words



Difficult to implement correctly in the general case

Modeling morphology and phonology

epäjärjestelmällistyttämättömyydelläänsäkäänköhän

Actual single Finnish word (not a compound!) ‘perhaps even because of his/her/it not having an ability to not generalize herself/himself/itself’ (maybe)

Grammatically correct, semantics is elusive, akin to ‘colorless green ideas sleep furiously’

Highly agglutinative languages like this have an astronomical number of “possible words”, even without considering neologisms

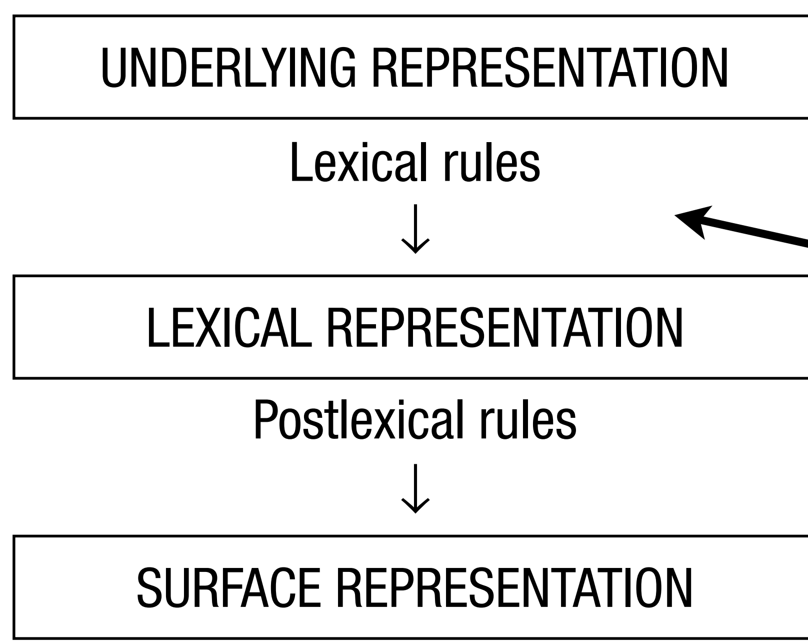
Linguistics: a model of word production

epäjärjestelmällistyttämättömyydelläänsäkäänköhän

Modeled by a step-by-step generative process:

‘un’+‘system‘ +‘ize’

epä+järjestelmä+lis+...



put morphemes together

phonemes and morphemes change when they are conjoined, modeled by phonological rules.

epäjärjestelmällistyttämättömyydelläänsäkäänköhän

“Generative” word model

in+possible+ity

(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

"Generative" word model

in+possible+ity

change n to m before p
(nasal assimilation)

im+possible+ity

(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

(2) Apply sound change rules + orthographic rules

"Generative" word model

(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

(2) Apply sound change rules + orthographic rules

in+possible+ity

change n to m before p (nasal assimilation)

im+possible+ity

ble+ity > bility

im+possibility

remove boundaries

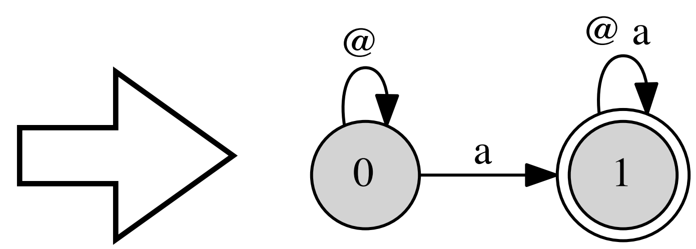
impossibility

Four tricks to model this

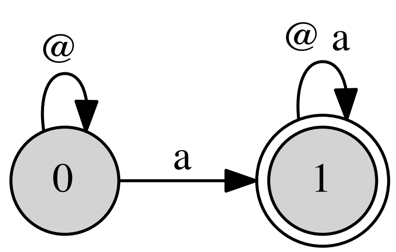
(1) Extended operators (Booleans, replacements)

(2) Use alphabet independent algorithms

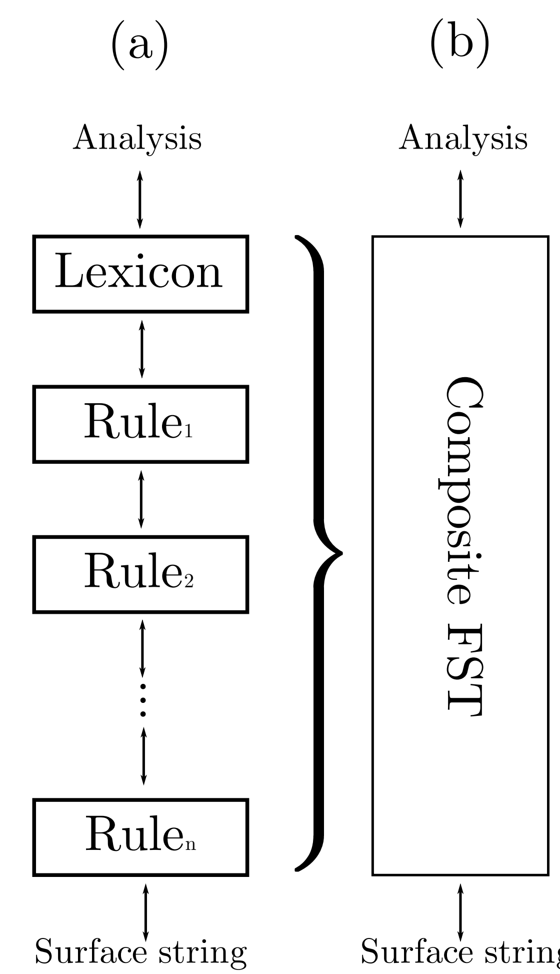
Σ^* a Σ^*



(3) Treat automata as "repeating transducers" ("everything is a transducer")



(repeat every word that contains at least one a)



(4) Model lexicon as an FST (which may just repeat words)

"Generative" word model

Lexicon + morphology

in+possible+ity

change n to m before p (nasal assimilation)

im+possible+ity

ble+ity → bility

im+possibility

remove boundaries

impossibility

(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

(2) Apply sound change rules + orthographic rules

"Generative" word model

Lexicon + morphology

in+possible+ity

n → m / _ + p

im+possible+ity

ble+ity → bility

im+possibility

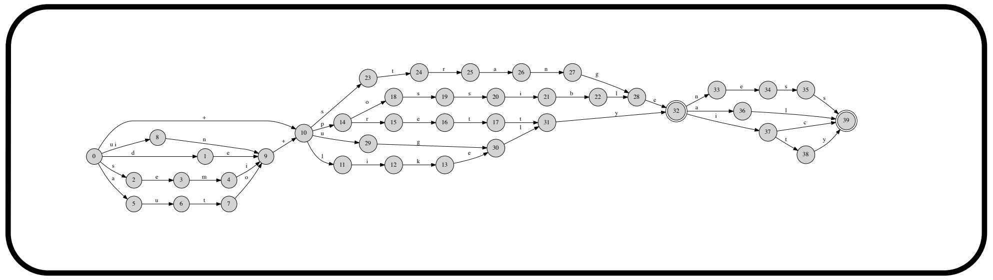
+ → 0

impossibility

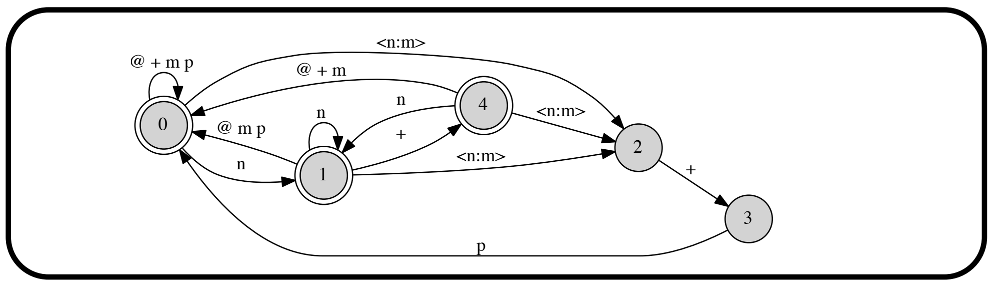
(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

(2) Apply sound change rules + orthographic rules

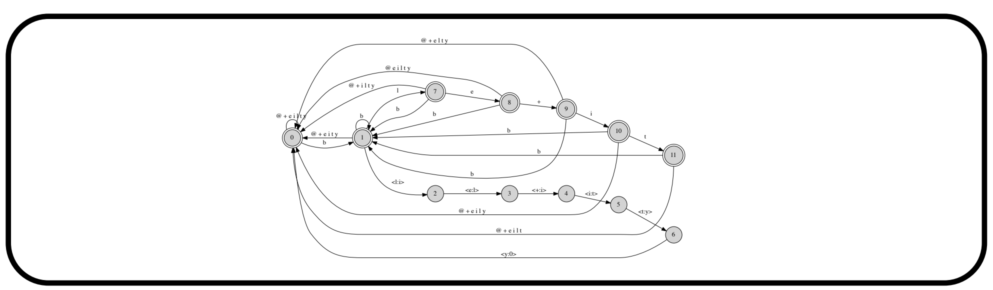
“Generative” word model



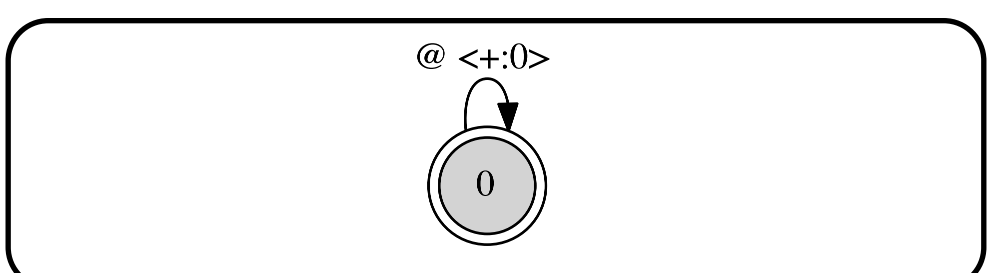
in+possible+ity



im+possible+ity



im+possibility



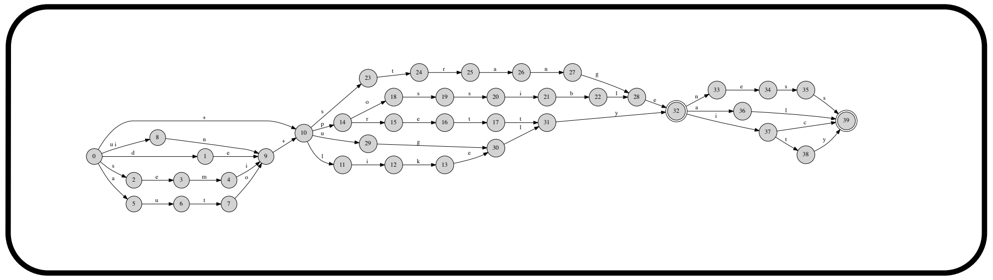
impossibility

(1) Pick morphemes from lexicon in right order and combinations (dictated by morphotactics)

(2) Apply sound change rules + orthographical rules

...then compose

Composition



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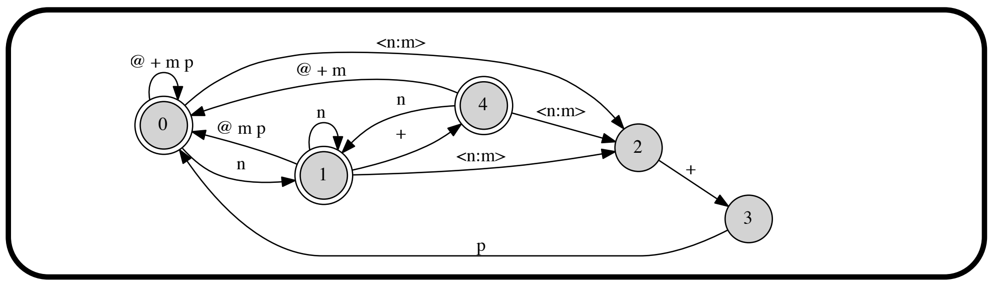

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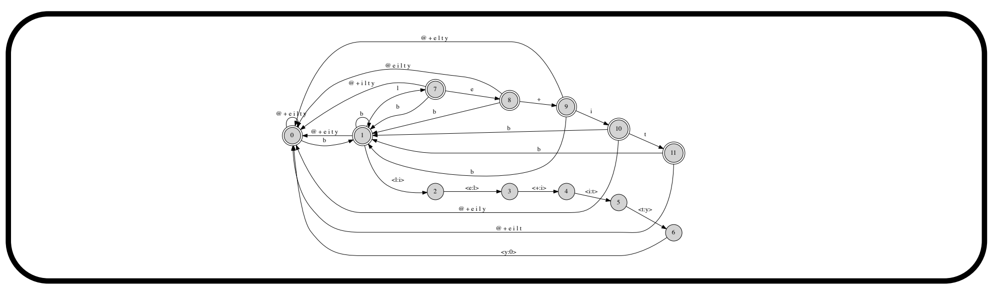
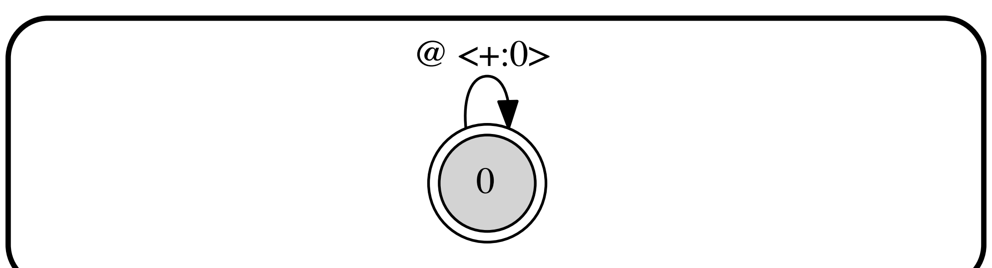
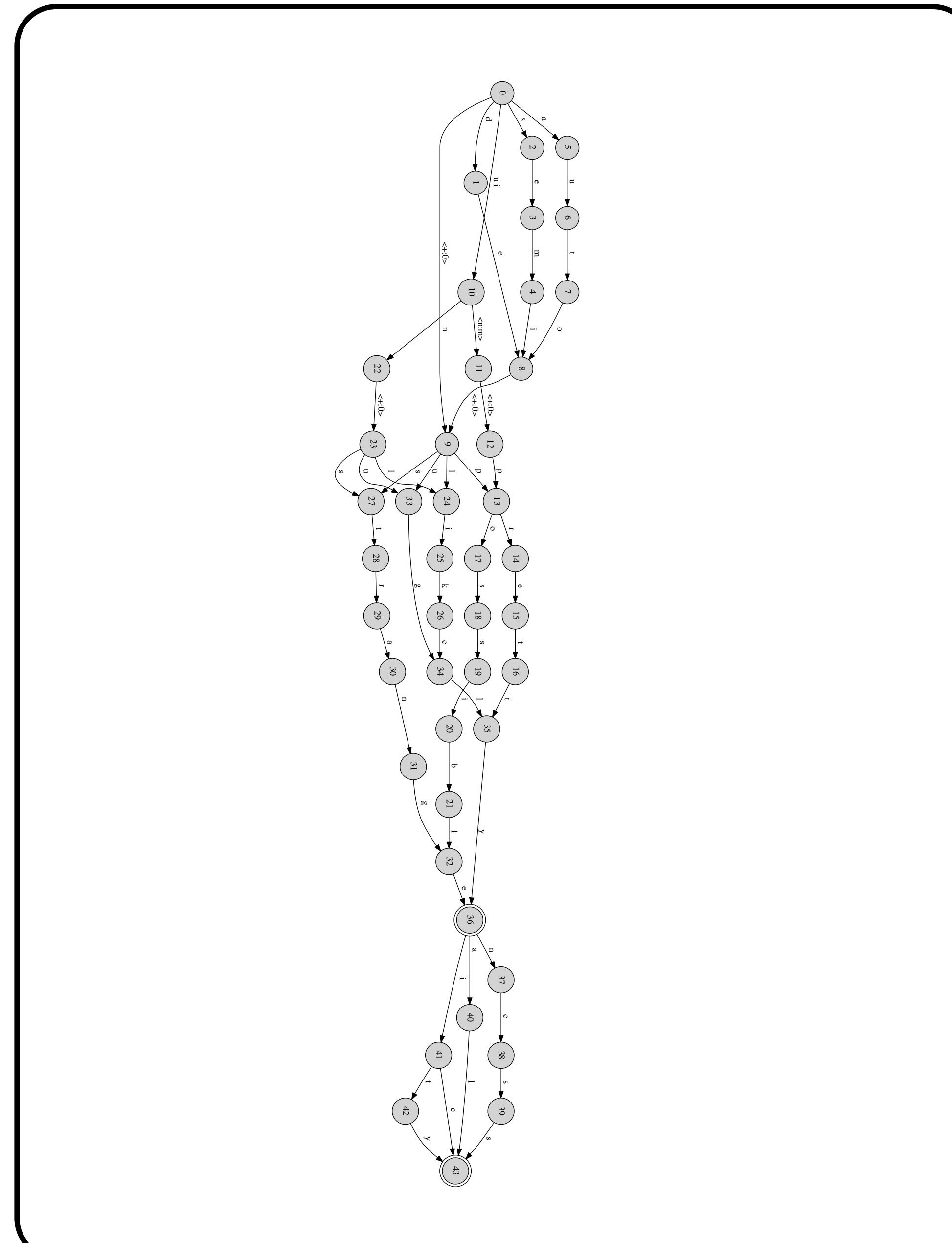


im+possibility



impossibility

in+possible+ity



impossibility

Adding grammatical information

We'd like to be able to get parses with grammatical information:

impossibilities => NEG+possible+ity+NOUN+PLURAL

vs.

in+possible+ity+s

Adding grammatical information

We'd like to be able to get parses with grammatical information:

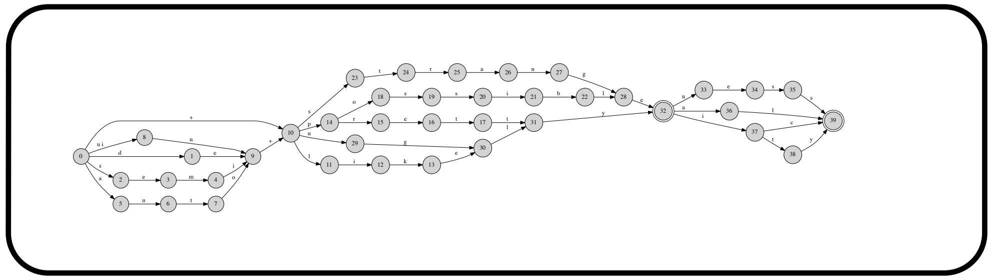
impossibilities => NEG+possible+ity+NOUN+PLURAL

vs.

in+possible+ity+s

Solution: make lexicon a transduction:

IN: NEG+possible+ity+NOUN+PLURAL

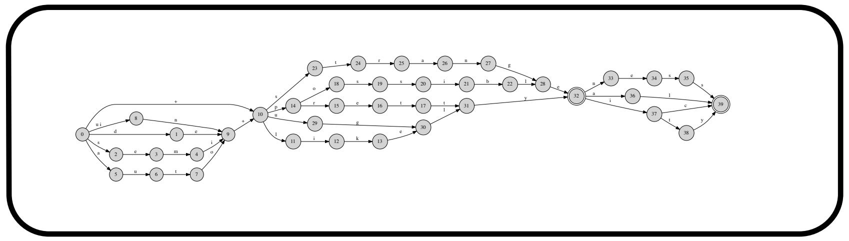


Lex. transducer

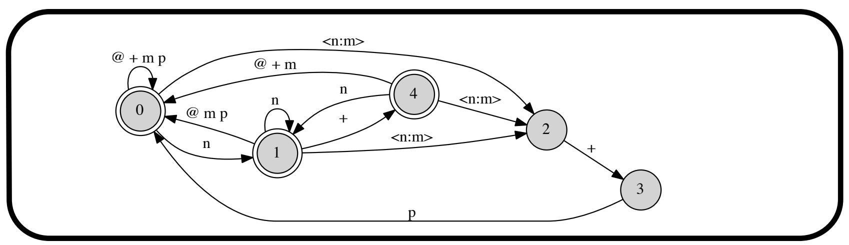
OUT: in+possible+ity+s

Composition

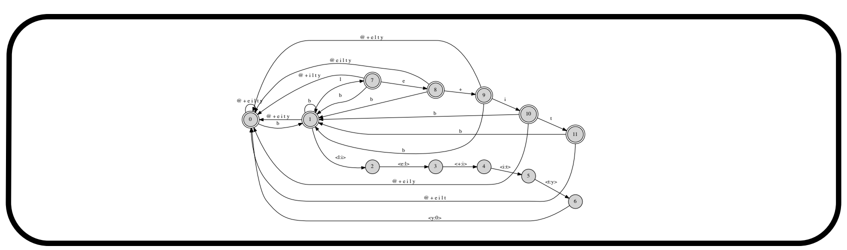
NEG+possible+ity+NOUN+PLURAL



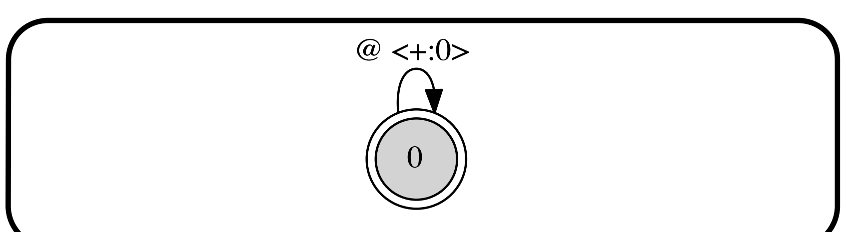
in+possible+ity+s



im+possible+ity+s



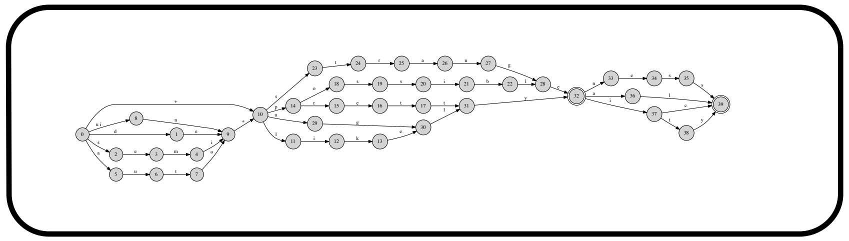
im+possibility+s



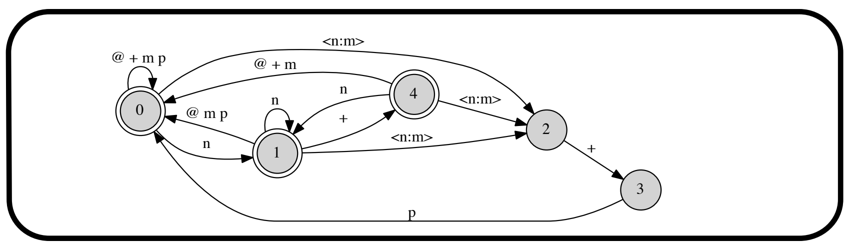
impossibilities

Composition

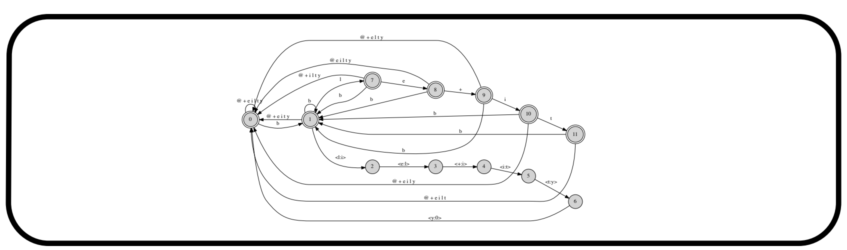
NEG+possible+ity+NOUN+PLURAL



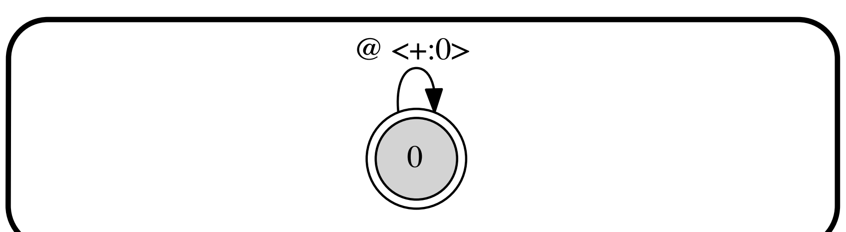
in+possible+ity+s



im+possible+ity+s

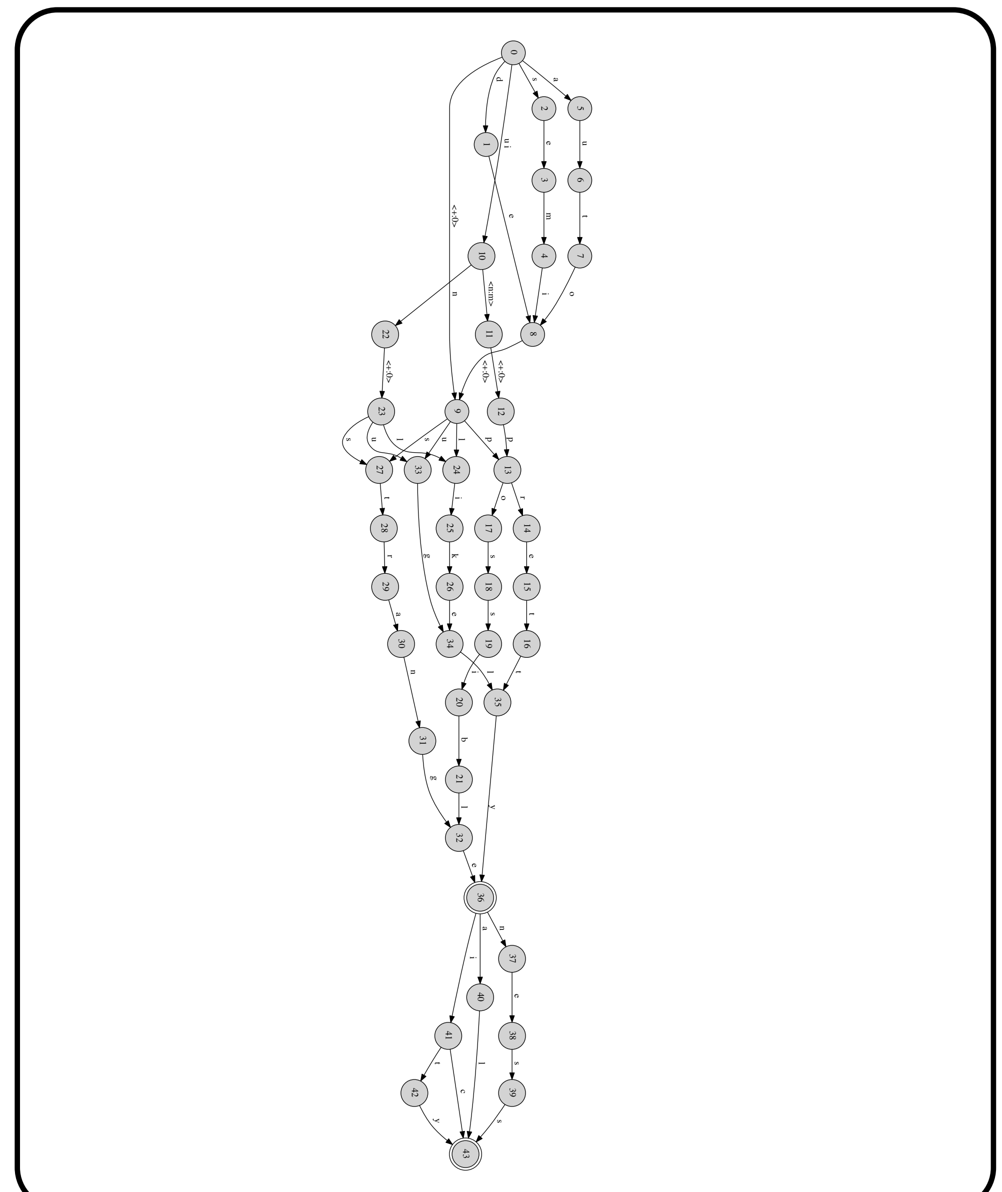


im+possibility+s



impossibilities

NEG+possible+ity+NOUN+PLURAL



impossibility

Compilers

Several finite-state compilers available to do the hard work

- Xerox xfst (http://www.fsmbook.com)
- SFST (https://code.google.com/p/cistern/wiki/SFST)
- HFST (http://hfst.sf.net)
- OpenFST (http://www.openfst.org)
- Foma (http://foma.googlecode.com)

Demo with foma